Rational coordination with no communication or conventions

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Abstract. We study pure coordination games where in every outcome, all players have identical payoffs, 'win' or 'lose'. We identify and discuss a range of 'purely rational principles' guiding the reasoning of rational players in such games and analyse which classes of coordination games can be solved by such players with no preplay communication or conventions. We observe that it is highly nontrivial to delineate a boundary between purely rational principles and other decision methods, such as conventions, for solving such coordination games.

1 Introduction

Pure coordination games ([11]), aka *games of common payoffs* ([12]), are strategic form games in which all players receive the same payoffs and thus all players have fully aligned preferences to coordinate in order to reach the best possible outcome for everyone. Here we study one-step *pure win-lose coordination games* (WLC games) in which all payoffs are either 1 (i.e., *win*) or 0 (i.e., *lose*).

Clearly, if players can communicate when playing a pure coordination game with at least one winning outcome, then they can simply agree on a winning strategy profile, so the game is trivialised. What makes such games non-trivial is the limited, or no possibility of communication before the game is presented to the players. In this paper we assume *no preplay communication*[1] *at all*, meaning that the players must make their choices by reasoning individually, without any contact with the other players before (or during) playing the game.

There are many natural real-life situations where such coordination scenarios occur. For example, (A) two cars driving towards each other on a narrow street such that they can avoid a collision by swerving either to the right or to the left. Or, (B) a group of n people who get separated in a city and they must each decide on a place where to get together ('regroup'), supposing they do not have any way of contacting each other.

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[1] Note that, unlike the common use of 'preplay communication' in game theory to mean communication before the given game is played, here we mean communication before the players are even presented with the game.

Even if no preplay communication is possible, players may still share some *conventions* ([8], [11], [18]) which they believe everyone to follow. In (A), a collision could be avoided by using the convention (or rule) that cars should always swerve to the right (or, to the left). In (B), everyone could go to a famous meeting spot in the city, e.g., the main railway station. Conventions need not be explicit agreements, but they can also naturally emerge as so-called *focal points*, for example. The theory of focal points, originating from Schelling [17], has been further developed in the context of coordination games, e.g. in [13], [20].

In this paper we assume that the players share no conventions, either. Thus, in our setting players play independently of each other. They can be assumed to come from completely different cultures, or even from different galaxies, for that matter. However, we assume that *it is common belief* among the players that:
(1) *every player knows the structure of the game*;
(2) *all players have the same goal*, viz. selecting together a winning profile;

Initially in this paper we will only assume *individual rationality*, i.e. that every player acts with the aim to win the game. Later we will assume in addition *common belief in rationality*, i.e. that every player is individually rational and that it is commonly believed amongst all players that every player is rational.

Our main objective is to analyse what kinds of reasoning can be accepted as 'purely rational' and what kinds of WLC games can be solved by such reasoning. Thus, we try to identify '*purely rational principles*' that *every* rational player ought to follow in *every* WLC game. We also study the hierarchy of such principles based on classes of WLC games that can be won by following different reasoning principles. It is easy to see that coordination by pure rationality is not possible in the example situations (A) and (B) above. However, we will see that there are many natural pure coordination scenarios in which it seems clear that rational players can coordinate successfully.

One of the principal findings of our study is that it is highly nontrivial to demarcate the "purely rational" principles from the rest[2]. Indeed, this seems to be an open-ended question and its answer depends on different background assumptions. Still, we identify a hierarchy of principles that can be regarded as rational and we also provide justifications for them. However, these justifications have varying levels of common acceptability and a more in-depth discussion would be needed to settle some of the issues arising there. Due to space constraints, a more detailed discussion on these issues is deferred to a follow-up work.

Coordination and rationality are natural and interesting topics that have been studied in various contexts in, e.g., [3], [6], [7], [8], [19]. We note the close conceptual relationship of the present study with the notion of *rationalisability* of strategies [2], [5], [15], which is particularly important in epistemic game theory. We also mention two recent relevant works related to logic to which the observations and results in the present paper could be directly applied: in [10], two-player coordination games were related to a variant of *Coalition Logic*[3], and

[2] Schelling shares this view on pure coordination games (see [17], pg. 283, ftn. 16).
[3] In fact, the initial motivation for the present work came from concerns with the semantics of Alternating time temporal logic ATL, extending Coalition Logic.

in [1], coordination was analysed with respect to the game-theoretic semantics of *Independence Friendly Logic*.

An extended version of this paper, containing more examples and technical details, is available as a companion technical report [9]. In addition to the theoretical work presented here, we have also run some empirical experiments on people's behaviour in certain WLC games. One of our tests can be accessed from the link given in the technical report [9].

2 Pure win-lose coordination games

2.1 The setting

A *pure win-lose coordination game* G is a strategic form game with n players $(1, \dots, n)$ whose available *choices* (*moves*, *actions*) are given by sets $\{C_i\}_{i \leq n}$. The set of winning *choice profiles* is presented by an n-ary *winning relation* W_G. For technical convenience and simplification of some definitions, we present these games as *relational structures* (see, e.g., [4]). A formal definition follows.

Definition 1. *An n-player* ***win-lose coordination game*** *(*WLC ***game****) is a relational structure $G = (A, C_1, \dots, C_n, W_G)$ where A is a finite domain of* ***choices****, each $C_i \neq \emptyset$ is a unary predicate, representing the choices of player i, s.t. $C_1 \cup \dots \cup C_n = A$, and W_G is an n-ary relation s.t. $W_G \subseteq C_1 \times \dots \times C_n$. Here we also assume that the players have pairwise disjoint choice sets, i.e., $C_i \cap C_j = \emptyset$ for every $i, j \leq n$ s.t. $i \neq j$. A tuple $\sigma \in C_1 \times \dots \times C_n$ is called a* ***choice profile*** *for G and the choice profiles in W_G are called* ***winning****.*

We use the following terminology for any WLC game $G = (A, C_1, \dots, C_n, W_G)$.

- Let $A_i \subseteq C_i$ for every $i \leq n$. The **restriction** of G to $(A_1, \dots, A_n)$ is the game $G \upharpoonright (A_1, \dots, A_n) := (A_1 \cup \dots \cup A_n,\ A_1, \dots, A_n,\ W_G \upharpoonright A_1 \times \dots \times A_n)$.
- For every choice $c \in C_i$ of a player i, the **winning extension of** c **in** G is the set $W_G^i(c)$ of all tuples $\tau \in C_1 \times \dots \times C_{i-1} \times C_{i+1} \times \dots \times C_n$ such that the choice profile obtained from τ by adding c to the i-th position is winning.
- A choice $c \in C_i$ of a player i is **(surely) winning**, respectively **(surely) losing**, if it is guaranteed to produce a winning (respectively losing) choice profile regardless of what choices the other player(s) make. Note that c is a winning choice iff $W_G^i(c) = C_1 \times \dots \times C_{i-1} \times C_{i+1} \times \dots \times C_n$. Similarly, c is a losing choice iff $W_G^i(c) = \emptyset$.
- A choice $c \in C_i$ is **at least as good as** (respectively, **better than**) a choice $c' \in C_i$ if $W_G^i(c') \subseteq W_G^i(c)$ (respectively, $W_G^i(c') \subsetneq W_G^i(c)$). A choice $c \in C_i$ is **optimal** for a player i if it is at least as good as any other choice of i.

Note that a choice $c \in C_i$ is better than a choice $c' \in C_i$ precisely when c *weakly dominates* c' in the usual game-theoretic sense (see e.g. [12], [16]), and a choice $c \in C_i$ is an optimal choice of player i when it is a weakly dominant choice. Note also that c *strictly dominates* c' (*ibid.*) if and only if c is surely winning and c' is surely losing. Thus, strict domination is a too strong concept in WLC games. Also the concept of *Nash equilibrium* is not very useful here.

Example 1. We present here a 3-player coordination story which will be used as a running example hereafter. The three robbers Casper, Jesper and Jonathan[4] are planning to quickly steal a cake from the bakery of Cardamom Town while the baker is out. They have two possible plans to enter the bakery: either (a) to break in through the front door or (b) to sneak in through a dark open basement. For (a) they need a *crowbar* and for (b) a *lantern*. The baker keeps the cake on top of a high cupboard, and the robbers can only reach it by using a *ladder*.

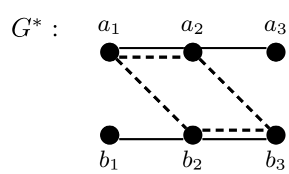
When approaching the bakery, Casper is carrying a crowbar, Jesper is carrying a ladder and Jonathan is carrying a lantern. However, the robbers cannot agree whether they should follow plan (a) or plan (b). While the robbers are quarreling, suddenly Constable Bastian appears and the robbers all flee to different directions. After this the robbers have to individually decide whether to go to the front door (by plan (a)) or to the basement entrance (by plan (b)). They must do the right decision fast before the baker returns.

The scenario we described here can naturally be modeled as a WLC game. We relate Casper, Jesper and Jonathan with players 1, 2 and 3, respectively. Each player i has two choices a_i and b_i which correspond to either going to the front door or to the basement entrance, respectively. The robbers succeed in obtaining the cake if both Casper and Jesper go to the front door (whence it does not matter what Jonathan does). Or, alternatively, they succeed if both Jonathan and Jesper go to the basement (whence the choice of Casper is irrelevant). Hence this coordination scenario corresponds to the following WLC game $G^* = (\{a_1, b_1, a_2, b_2, a_3, b_3\}, C_1, C_2, C_3, W_{G^*})$, where for each player i, $C_i = \{a_i, b_i\}$ and $W_{G^*} = \{(a_1, a_2, a_3), (a_1, a_2, b_3), (a_1, b_2, b_3), (b_1, b_2, b_3)\}$. (For a graphical presentation of this game, see Example 2 below.)

2.2 Presenting WLC games as hypergraphs

The n-ary winning relation W_G of an n-player WLC game G defines a *hypergraph* on the set of all choices. We give visual presentations of hypergraphs corresponding to WLC games as follows: The choices of each player are displayed as columns of nodes starting from the choices of player 1 on the left and ending with the column with choices of player n. The winning relation consists of lines that go through some choice of each player[5]. This kind of graphical presentation of a WLC game G will be called a *game graph (drawing) of G*.

Example 2. The WLC game G^* in Example 1 has the following game graph:



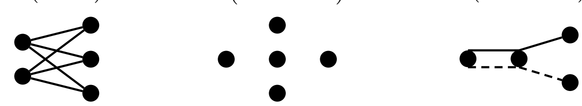
[4] This example is based on the children's book *When the Robbers Came to Cardamom Town* by Thorbjørn Egner, featuring the characters Casper, Jesper and Jonathan.

[5] In pictures these lines can be drawn in different styles or colours, to tell them apart.

We now introduce a uniform notation for certain simple classes of WLC games. Let $k_1, \dots, k_n \in \mathbb{N}$.

- $G(k_1 \times \cdots \times k_n)$ is the n-player WLC game where the player i has k_i choices and the winning relation is the *universal relation* $C_1 \times \cdots \times C_n$.
- $G(\overline{k_1 \times \cdots \times k_n})$ is the n-player WLC game where the player i has k_i choices and the winning relation is the *empty relation*. Some examples:

$G(2 \times 3)$ $G(\overline{1 \times 3 \times 1})$ $G(1 \times 1 \times 2)$

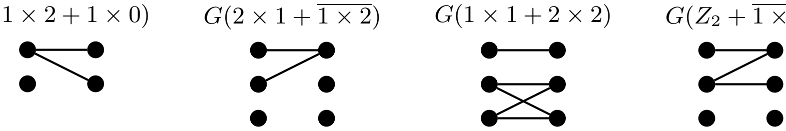


- Let $k \in \mathbb{N}$. We write $G(Z_k)$ for the *2-player* WLC game in which both players have k choices and the winning relation forms a single path that goes through all the choices (see below for an example). Similarly, $G(O_k)$, where $k \geq 2$, denotes the 2-player WLC game where the winning relation forms a $2k$-cycle that goes through all the choices. These are exemplified by the following:

$G(Z_2)$ $G(Z_3)$ $G(O_2) = G(2 \times 2)$ $G(O_3)$

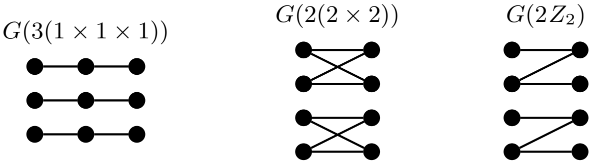
- Suppose that $G(A)$ and $G(B)$ have been defined, both having the same number of players. Then $G(A + B)$ is the *disjoint union* of $G(A)$ and $G(B)$, i.e., the game obtained by assigning to each player a disjoint union of her choices in $G(A)$ and $G(B)$, and where the winning relation for $G(A+B)$ is the union of the winning relations in $G(A)$ and $G(B)$. Some examples:

$G(1 \times 2 + 1 \times 0)$ $G(2 \times 1 + \overline{1 \times 2})$ $G(1 \times 1 + 2 \times 2)$ $G(Z_2 + \overline{1 \times 1})$



- Let $m \in \mathbb{N}$. Then $G(mA) := G(A + \cdots + A)$ (m times). Examples:

$G(3(1 \times 1 \times 1))$ $G(2(2 \times 2))$ $G(2Z_2)$



- Recall our "regrouping scenario" (B) from the introduction. If there are n people in the group and there are m possible meeting spots in the city, then the game is of the form $G(m(1^n))$, where $1^n := 1 \times \cdots \times 1$ (n times).

2.3 Symmetries of WLC games and structural protocols

A **protocol** is a mapping Σ that assigns to every pair (G, i), where G is a WLC game and i a player in G, a nonempty set $\Sigma(G, i) \subseteq C_i$ of choices. Thus a protocol gives global nondeterministic strategy for playing any WLC game in the role of any player. Intuitively, a protocol represents a global mode of acting in any situation that involves playing WLC games. Hence, protocols can be informally regarded as global "reasoning styles" or "behaviour modes". Thus, a protocol can also be identified with an agent who acts according to that protocol in all situations that involve playing different WLC games in different player roles.

Assuming a setting based on pure rationality with no special conventions or preplay communication, a protocol will only take into account the *structural properties* of the game and its winning relation. Thus the names of the choices and the names (or ordering) of the players should be of no relevance. In this section we make this issue precise. (For more details and examples, see [9].)

Definition 2. *An isomorphism*[6] *between games G and G' is called a **choice-renaming**. An automorphism of G is called a **choice-renaming of** G.*

Let $G = (A, C_1, \ldots, C_n, W_G)$ be a WLC *game. For a player i, we say that the choices $c, c' \in C_i$ are **i-equivalent**, denoted by $c \simeq_i c'$, if there is a choice-renaming of G that maps c to c'. For each $i \leq n$, the relation $\simeq_i$ is an equivalence relation on the set C_i. We denote the equivalence class of $c \in C_i$ by $[\![c]\!]_i$.*

Definition 3. *Consider n-player* WLC *games $G = (A, C_1, \ldots, C_n, W_G)$ and $G' = (A, C'_1, \ldots, C'_n, W'_G)$. A permutation $\beta : \{1, ..., n\} \to \{1, ..., n\}$ is called a **player-renaming** between G and G' if the following conditions hold:*

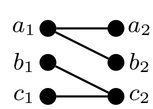
(1) $C_{\beta(i)} = C'_i$ *for each* $i \leq n$.
(2) $W'_G = \{\, (c_{\beta(1)}, \ldots, c_{\beta(n)}) \mid (c_1, \ldots, c_n) \in W_G \,\}$.

If there is a player-renaming between two WLC games, the games are essentially the same, the only difference being the ordering of the players.

Definition 4. *Consider* WLC *games G and G'. A pair (β, π) is a **full renaming** between G and G' if there is a* WLC *game G'' such that β is a player-renaming between G and G'' and π is a choice-renaming between G'' and G'. If $G = G'$, we say that (β, π) is a **full renaming of** G. We say that choices $c \in C_i$ and $c' \in C_j$ in the same game are **structurally equivalent**, denoted by $c \sim c'$, if there is a full renaming (β, π) of G such that $\beta(i) = j$ and $\pi(c) = c'$. It is quite easy to see that $\sim$ is an equivalence relation on the set A of all choices. We denote the equivalence class of a choice c by $[c]$.*

Example 3. Consider a WLC game of the form $G(1 \times 2 + 2 \times 1)$.

$G(1 \times 2 + 2 \times 1)$**:**



[6] Isomorphism is defined as usual for relational structures (see, e.g., [4]).

It is easy to see that $\simeq_1$ has the equivalence classes $\{a_1\}$ and $\{b_1, c_1\}$, and similarly, $\simeq_2$ has the equivalence classes $\{c_2\}$ and $\{a_2, b_2\}$. Furthermore, $\sim$ has the equivalence classes $\{a_1, c_2\}$ and $\{b_1, c_1, a_2, b_2\}$. Likewise, in the game G^* from Example 1 the relation $\sim$ has the equivalence classes $\{a_1, b_3\}$, $\{b_1, a_3\}$, $\{a_2, b_2\}$.

We say that a protocol Σ is **structural** if it is "indifferent" with respect to full renamings, which means that, given any WLC games G, G' for which there exists a full renaming (β, π) between G and G', for any i and any choice $c \in C_i$, it must hold that $c \in \Sigma(G, i)$ iff $\pi(c) \in \Sigma(G', \beta(i))$. Intuitively, this reflects the idea that when following a structural protocol, one acts independently of the names of choices and names (or ordering) of player roles.

It is worth noting that if we considered a framework where WLC games were presented so that the names of the choices and players could be used to define an ordering (of the players and their choices), things would trivialize because it would be easy to win all games by the prenegotiated agreement to always choose the lexicographically least tuple from the winning relation.

3 Purely rational principles in WLC games

By a **principle** we mean any nonempty class of protocols. Intuitively, these are the protocols "complying" with that principle. If protocols are regarded as "reasoning styles" (or "behaviour modes"), then principles are *properties* of such reasoning styles (or behaviour modes). Principles that contain only structural protocols are called **structural principles.**

A player i **follows a principle P** in a WLC game G if she plays according to some protocol in P. We are mainly interested in structural principles which describe "purely rational" reasoning that involves neither preplay communication nor conventions and which are rational to follow in *every* WLC game. Such principles will be called **purely rational principles**. Intuitively, purely rational principles should always be followed by all rational players. Consider:

$$\mathrm{P}_1 := \{\Sigma \mid \Sigma(G, i) \text{ does not contain any surely losing choices when } W_G \neq \emptyset\},$$
$$\mathrm{P}_2 := \{\Sigma \mid \Sigma(G, i) \text{ contains all choices } c \in C_i \text{ such that } |W_G^i(c)| \text{ is a prime number. If there are no such choices, } \Sigma(G, i) = C_i.\}.$$

If player i follows P_1, then she always uses some protocol which does not select surely losing choices, if possible. This seems a principle that any rational agent would follow. If player i follows P_2, then she always plays choices whose degree (in the game graph) is a prime number, if possible. Note that both principles are structural, but P_1 can be seen as a purely rational principle, while P_2 seems arbitrary; it could possibly be some seemingly odd convention, for example.

We say that a **principle P solves** a WLC game G (or G **is P-solvable**), if G is won whenever every player follows some protocol that belongs to P. Formally, this means that $\Sigma_1(G, 1) \times \cdots \times \Sigma_n(G, n) \subseteq W_G$ for all protocols $\Sigma_1, \ldots, \Sigma_n \in \mathrm{P}$. The class of all P-solvable games is denoted by $s(\mathrm{P})$.

In this paper we try to identify (a hierarchy of) principles that can be considered to be purely rational and analyse the classes of games that they solve.

3.1 Basic individual rationality

Hereafter we describe principles by the properties of protocols that they determine. We begin by considering the case where players are individually rational, but there is no common knowledge about this being the case. It is safe to assume that any individually rational player would follow at least the following principle.

Fundamental individual rationality (FIR):
Never play a strictly dominated choice.[7]

As noted before, strict domination is a very weak concept with WLC games. Following FIR simply means that a player should never prefer a losing choice to a winning one. Therefore FIR is a very weak principle that can solve only some quite trivial types of games such as $G(1 \times 2 + 1 \times 0)$. In general, FIR-solvable games have a simple description: at least one of the players has (at least one) winning choice, and all non-winning choices of that player are losing. FIR has two natural strengthenings which can be considered purely rational:

1. **Non-losing principle (NL):** Never play a losing choice, if possible.
2. **Sure winning principle (SW):** Always play a winning choice, if possible.

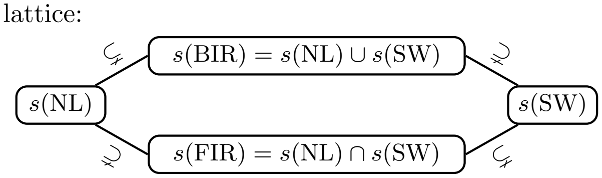
Since losing choices cannot be winning choices, these principles can naturally be put together (by taking the intersection of these principles):

Basic individual rationality (BIR): NL $\cap$ SW.

When following BIR, a player plays a winning choice if she has one, and else she plays a non-losing choice. We make the following observations. (For a more detailed justification of these claims, see the technical report [9].)

1. NL and SW do not imply each other and neither of them follows from FIR. This can be seen by the following examples.
   - The game $G(1 \times 1 + \overline{1 \times 1})$ is NL-solvable but not SW-solvable.
   - The game $G(Z_2)$ is SW-solvable but not NL-solvable.
2. FIR-solvable games are solvable by *both* SW and NL.
3. Every BIR-solvable game is *either* NL or SW-solvable.

Therefore we can see that the sets of games solvable by FIR, NL, SW, BIR form the following lattice:



SW-solvable and NL-solvable games have simple descriptions: In SW-solvable games, at least one player has a surely winning choice. In NL-solvable games, the winning relation forms a nonempty *Cartesian product* between all non-losing choices. BIR-solvable games have (at least) one of these two properties.

[7] Recall, that a choice a is strictly dominated by a choice b if the choice b guarantees a strictly higher payoff than the choice a in every play of the game (see e.g. [12], [16]).

3.2 Common beliefs in rationality and iterated reasoning

In contrast to individual rationality, collective rationality allows players to make assumptions on each other's rationality. Let P be a (purely rational) principle. When *all players believe that everyone follows* P, they can reason as follows:

($\star$) Suppose that by following P each player i must play a choice from $A_i \subseteq C_i$ (that is, A_i is the smallest set such that $\Sigma(G, i) \subseteq A_i$ for every $\Sigma \in \mathrm{P}$). By this assumption, the players may collectively assume that the game that is played is actually $G' := G \upharpoonright (A_1, \ldots, A_n)$, and therefore all P-compliant protocols should only prescribe choices in G'.

If players have *common belief* in P being followed, then the reasoning ($\star$) above can be repeated for the game G' and this iteration can be continued until a fixed point is reached. By $\mathsf{cir}(\mathrm{P})$ we denote the principle of **collective iterated reasoning of P** which prescribes that P is followed in the reduced game obtained by the iterated reasoning of ($\star$). Since every iteration of ($\star$) only reduces the players' sets of acceptable choices (yet, keeps them nonempty), it is easy to see that $s(\mathrm{P}) \subseteq s(\mathsf{cir}(\mathrm{P}))$ for any principle P (see [9] for more details.)

When considering principles of *collective* rationality, we will apply collective iterated reasoning. It may be debated whether such reasoning counts as purely rational, so a question arises: if P is a purely rational principle, is $\mathsf{cir}(\mathrm{P})$ always purely rational as well? For the lack of space we will not discuss this issue here. We note, however, the extensive literature relating common beliefs and knowledge with individual and collective rationality, see e.g. [5], [11], [14], [21].

3.3 Basic collective rationality

Here we extend individually rational principles of Section 3.1 by adding common belief in the principles (as described in Section 3.2) to the picture. We first analyse what happens with principles NL and SW. It is easy to see that the collective iterated reasoning of NL reaches a fixed point in a single step by simply removing the losing choices of every player. Hence $s(\mathrm{NL}) = s(\mathsf{cir}(\mathrm{NL}))$. Collective iterated reasoning of SW also reaches a fixed point in a single step by eliminating all non-winning choices of every player who has a winning choice. But if even one player has a winning choice, then the game is already SW-solvable. Therefore $s(\mathrm{SW}) = s(\mathsf{cir}(\mathrm{SW}))$.

However, assuming common belief in BIR, some games which are not BIR-solvable may become solvable. See the following example.

Example 4. The game $G(Z_2 + \overline{1 \times 1})$ cannot be solved with NL or SW. However, if the players can assume that neither of them selects a losing choice (by NL) and eliminate those choices from the game, then they (both) have a winning choice in the reduced game and can win in it by SW.

Thus, we define the following principle:

Basic collective rationality (BCR): $\mathsf{cir}(\mathrm{BIR})$.

The above example shows that $s(\mathrm{BIR}) \subsetneq s(\mathrm{BCR})$, i.e. BCR is *stronger* than BIR. The games solvable by BCR have the following characterisation: *after removing all surely losing choices of every player, at least one of the players has a surely winning choice.* It is worth noting that common belief in SW is not needed for solving games with BCR because a *single* iteration of cir(NL) suffices. Thus, players could solve BCR-solvable games simply by following BIR and believing that everyone follows NL. We also point out that the principle BCR is equivalent to the principle applied in [10] for Strategic Coordination Logic.

3.4 Principles using optimal choices

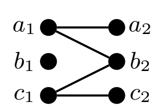
If a rational player has optimal choices (that are at least as good as all other choices), it is natural to assume that she selects such a choice.

Individual optimal choices (IOC): Play an optimal choice, if possible.

Example 5. Recall the WLC game G^* from Example 1. For Casper (who is carrying the crowbar) it is a better choice to go to the front door than to the basement. Likewise, for Jonathan (who is carrying the lantern) it is a better choice to go to the basement than to the front door. Therefore the choice a_1 is (the only) optimal choice for player 1 and b_3 is (the only) optimal choice for the player 3. The player 2 (Jesper) does not have any optimal choices, but if both 1 and 3 play their optimal choices, then the game is won regardless of the choice of 2. Therefore, the game G^* is solvable with IOC. But since no player has winning or losing choices in this game, it is easy to see that it is not BCR-solvable.

By the description of BIR-solvable games, it is easy to see that they are IOC-solvable. We will show that IOC is *incomparable* with BCR (in terms of their sets of solvable games). As explained above, the game G^* is IOC-solvable but not BCR-solvable. Furthermore, the following BCR-solvable game G_Σ is not IOC-solvable since player 1 does not have any optimal choices and so might end up playing a losing choice.

G_Σ:



In order to avoid pathological cases like this we can add NL to IOC.

Improved basic individual rationality (BIR$^+$): $\mathrm{IOC} \cap \mathrm{NL}$

This principle is stronger than BCR (see [9]) even though it is based only on *individual* reasoning. We now consider the collective version of IOC:

Collective optimal choices (COC): cir(IOC)

We can show that COC is stronger than BIR$^+$ and therefore also stronger than BCR (see [9]). Finally, observe that in a 2-player WLC game G where $W_G \neq \emptyset$ *the only optimal choices are those that are winning against all non-losing choices of the other player.* Therefore, in the special case of 2-player WLC games, it is easy to see that the hierarchy collapses as $s(\mathrm{BCR}) = s(\mathrm{BIR}^+) = s(\mathrm{COC})$.

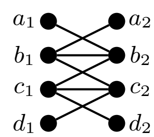
3.5 Elimination of weakly dominated choices

Usually in game-theory, rationality is associated with the elimination of strictly or weakly dominated strategies. As noted in Section 3.1, *strict* domination is a too strong concept for WLC games. Weak domination, on the other hand, gives the following principle when applied individually.

Individually rational choices (IRC): Do not play a choice a when there is a better choice b available, i.e., if $W_G^i(a) \subsetneq W_G^i(b)$, then i does not play a.

Note that by this definition, when a player follows IRC, she also follows NL and IOC, and therefore $s(\mathrm{BIR}^+) \subseteq s(\mathrm{IRC})$. The inclusion is, in fact, proper since the following WLC game $G_{\#}$ is solvable with IRC but not with BIR^+.

$G_{\#}$:



We show in [9] that IRC is incomparable with COC. However, based on the observations above, in the 2-player case $s(\mathrm{COC}) = s(\mathrm{BIR}^+) \subsetneq s(\mathrm{IRC})$.

We next assume common belief in IRC. As commonly known (see e.g. [14]), *iterated elimination of weakly dominated strategies* eventually stabilises in some reduced game but different elimination orders may produce different results. However, when applying cir(IRC), the process will stabilise to a unique reduced game since all weakly dominated choices are always removed *simultaneously*. By following the next principle, players will play a choice within this reduced game.

Collective rational choices (CRC): cir(IRC)

For example, $G(Z_3)$ is not solvable with IRC, but can be solved with CRC by doing two collective iterations of IRC. Thus $s(\mathrm{IRC}) \subsetneq s(\mathrm{CRC})$. This observation can be generalized as follows: to solve a game of the form $G(Z_n)$, the players need $n-1$ iterations of IRC. Therefore different numbers of iterations of IRC form a proper hierarchy of CRC-solvable 2-player WLC games within $s(\mathrm{CRC})$.

3.6 Symmetry-based principles

By only following the concept of rationality from game-theory, one could argue that CRC reaches the border of purely rational principles. However, we now define more principles which are incomparable with CRC but can still be regarded as purely rational. These principles are based on *symmetries* in WLC games and the assumption that players follow only structural protocols is central here.

We begin with auxiliary definitions. We say that a choice profile $(c_1, \ldots, c_n)$ **exhibits a bad choice symmetry** if $[\![c_1]\!]_1 \times \cdots \times [\![c_n]\!]_n \not\subseteq W_G$ (recall Definition 2), and that a choice c **generates a bad choice symmetry** if σ_c exhibits bad choice symmetry for *every* choice profile σ_c that contains c.

Elimination of bad choice symmetries (ECS):
Never play choices that generate a bad choice symmetry, if possible.

Why should this principle be considered rational? Suppose that a player i plays a choice c_i which generates a bad choice symmetry. It is now possible to win only if some tuple $(c_1, \ldots, c_{i-1}, c_i, c_{i+1}, \ldots, c_n) \in W_G$ is eventually chosen. However, the players have *exactly the same reason* (based on structural principles) to play so that any other tuple in $[\![c_1]\!]_1 \times \cdots \times [\![c_n]\!]_n$ is selected, and such other tuple may possibly be a losing one since $[\![c_1]\!]_1 \times \cdots \times [\![c_n]\!]_n \not\subseteq W_G$.

Example 6. Here is a typical example of using ECS. Suppose that the game graph of G has two (or more) connected components that are isomorphic to each other. Since no player can see a difference between those components, all players should avoid playing choices from them. With this reasoning, games like $G(1\times 1+2(1\times 2))$ can be solved. Note that this game is not CRC-solvable since no player has any weakly dominated choices.

While ECS only considers symmetries between similar choices, the next principle takes symmetries *between players* into account. Consider a choice profile $\boldsymbol{c} = (c_1, \ldots, c_n)$ and let $S_i^p(\boldsymbol{c}) := \{c_i\} \cup (C_i \cap \bigcup_{j\neq i}[c_j])$ for each i (recall Definition 4). We say that $(c_1, ..., c_n)$ **exhibits a bad player symmetry** if $S_1^p(\boldsymbol{c}) \times \cdots \times S_n^p(\boldsymbol{c}) \not\subseteq W_G$ and a choice c **generates a bad player symmetry** if σ_c exhibits a bad player symmetry for every choice profile σ_c that contains c.

Elimination of bad player symmetries (EPS):
Never play choices that generate bad player symmetries, if possible.

Here the players assume that all players reason similarly, or alternatively, each player wants to play so that she would at least coordinate with herself in the case she was to use her protocol to make a choice in each player role of a WLC game. Suppose that the players have some reasons to select a choice profile $(c_1, \ldots, c_n)$. Now, if there are players $i \neq j$ and a choice $c'_j \in C_j$ such that $c'_j \sim c_i$, then the player j should have the same reason to play c'_j as i has for playing c_i. Hence, if the players have their reasons to play $(c_1, \ldots, c_n)$, they should have the same reasons to play any choice profile in $S_1^p(\boldsymbol{c})\times\cdots\times S_n^p(\boldsymbol{c})$. Winning is not guaranteed if $S_1^p(\boldsymbol{c}) \times \cdots \times S_n^p(\boldsymbol{c}) \not\subseteq W_G$.

Example 7. Consider EPS in the case of a two-player game WLC game G. If for a given choice $c \in C_1$, there is a structurally equivalent choice $c' \in C_2$ such that $(c, c') \notin W_G$, then by following EPS, player 1 does not play the choice c (and likewise player 2 does not play the choice c'). With this kind of reasoning, some CRC-unsolvable games like $G(1 \times 1 + 1 \times 2 + 2 \times 1)$ become solvable.

Note also that the game G^* (recall Example 1) is EPS-solvable since both choices b_1 and a_3 generate a bad player symmetry.

Finally, we introduce a principle that takes both types of symmetries into account. For a choice profile $\boldsymbol{c} = (c_1, ..., c_n)$ let $S_i(\boldsymbol{c}) := C_i \cap \bigcup_j [c_j]$ for each i. We say that $(c_1, ..., c_n)$ **exhibits a bad symmetry** if $S_1(\boldsymbol{c}) \times \cdots \times S_n(\boldsymbol{c}) \not\subseteq W_G$, and a choice c **generates a bad symmetry** if σ_c exhibits a bad symmetry for every choice profile σ_c that contains c.

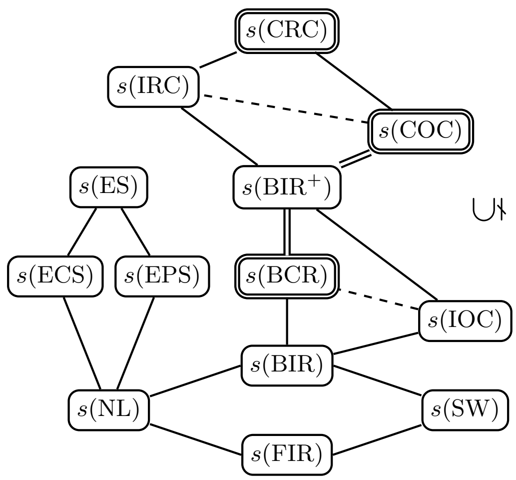
Elimination of bad symmetries (ES):
Never play choices that generate bad symmetries, if possible.

It is easy to show that ECS and EPS are not comparable and that they are both weaker than ES. Furthermore, all symmetry based principles can clearly solve NL-solvable games, but they are incomparable with SW and all the stronger principles. For proofs of these claims and further examples, see [9].

In a follow-up work we will address questions about compatibility of the symmetry principles ECS and EPS with each other and with the other principles considered so far, in particular with CRC which is the strongest of them.

3.7 Hierarchy of the principles presented so far

The partially ordered diagram below presents the hierarchy of solvable games with the principles we have presented in this paper. The principles that only use individual reasoning have normal frames and the ones that use collective reasoning have double frames.



- Normal lines represent proper inclusions in both the general *and* 2-player case.
- *Double* lines represent proper inclusions in the general case. In the 2-player case there is an identity.
- *Dashed* lines represent proper inclusions in the 2-player case. In the general case the two sets are not comparable.

3.8 Beyond the limits of pure rationality

How far can we go up the hierarchy of rational principles? This seems a genuinely difficult question to answer. We now mention—without providing precise formal definitions—two structural principles for which it would seem somewhat controversial to claim them rational in our sense, but they are definitely meaningful and natural nevertheless.

The first one is the **principle of probabilistically optimal reasoning (PR)**. Informally put, this principle prescribes to always play a choice that have as large winning extension as possible. These choices have the highest probability of winning, supposing that all the other players play randomly (but *not* if the others follow PR, too: consider e.g. $G(1 \times 2 + 2 \times 1)$).

With PR one can solve games like $G(1\times1+2\times2)$ that are unsolvable with all other principles presented here. However, in $G(1\times1+2\times2)$ one could also reason (perhaps less convincingly) that both players should pick their choices from the subgame $G(1\times1)$ since that is the '*simplest*' (and, also the only '*unique*') winning choice profile. We call this kind of reasoning the **Occam razor principle (OR)**. In fact, it generalises the idea of *focal point* [13], [17], [20].

Note that $G(1 \times 1 + 2 \times 2)$ can be won if both players follow PR or if both follow OR, but not if one follows PR while the other follows OR. Moreover, in this game it is impossible for a player to follow *both* PR and OR. Hence, *at least one* of these principles is not purely rational. Actually, it can be argued that *none of them* is purely rational. It is also interesting to note that following PR can violate the symmetry principles, as demonstrated by the game $G(2(2\times2)+1\times1)$.

3.9 Characterising structurally unsolvable games

So far we have characterised several principles with different levels of justification for being purely rational. It seems difficult to pinpoint a single strongest principle of pure rationality, but even if such a principle existed, certain games would nevertheless be unsolvable (assuming that purely rational principles must be *structural*). The simplest nontrivial example of such a game is $G(2(1 \times 1))$.

We now characterise the class of WLC games that are **structurally unsolvable**, i.e., *unsolvable by any structural principle*. We say that G is **structurally indeterminate** if all choice profiles in W_G exhibit a bad symmetry (recall the definition of the principle ES). For an example the game $G(1 \times 2 + 2 \times 1)$ is structurally indeterminate, whereas the game $G(1 \times 1 + 2 \times 2)$ is not.

Claim I. *No structural principle can solve a structurally indeterminate game.*

For a proof for this claim, see [9]. This characterisation is optimal in the sense that all games that are not structurally indeterminate, can be solved by some structural principle. This follows from the following even stronger claim.

Claim II. *There exists a protocol Σ such that the principle $\{\Sigma\}$ can solve all* WLC *games that are not structurally indeterminate.*

For a proof, see [9]. There are many games that are not structurally unsolvable, but in order to solve them, the players need to follow structural principles that seem arbitrary and certainly cannot be considered purely rational. We call such principles *structural conventions*. However, it is difficult to separate some rational principles from structural conventions. This and other related conceptual issues will be discussed in an extended version of this paper.

4 Concluding remarks

In this paper we have focused on scenarios where players look for choices that guarantee winning if a suitable rational principle is followed. But it is very natural to ask how players should act in a game which seems not solvable by any purely rational principle. If players cannot guarantee a win, it is natural to assume that they should at least try to maximize somehow their collective chances of winning, say, by considering protocols involving some probability distribution between their choices. Another natural extension of our framework is to consider non-structural principles based on limited preplay communication and use of various types of conventions. Also, studying pure dis-coordination games and combinations of coordination/dis-coordination are major lines for further work.

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