Coalition Games and Alternating Temporal Logics

(Corrected version: September 7, 2001)

Valentin Goranko
Department of Mathematics, Rand Afrikaans University
PO Box 524, Auckland Park 2006, Johannesburg, South Africa
E-mail: vfg@na.rau.ac.za

Abstract

We draw parallels between coalition game logics developed in [Pauly, 2000b], [Pauly, 2000c], and [Pauly, 2001] on one hand, and alternating-time temporal logics of computations introduced in [Alur et al, 97] on the other. In particular, we show equivalence of their semantics, embedding of coalition game logics into alternating-time temporal logic, and propose axiomatic systems for these logics.

1 Introduction

In this study we offer a comparative analysis of two independent recent logical enterprises: *coalition game logics* (see [Pauly, 2000b], [Pauly, 2000c],[Pauly, 2001]) and *alternating-time temporal logics* (see [Alur et al, 97]). These turn out to be intimately related, which is not surprising since both deal with essentially the same type of scenarios, viz. a *set of agents* (*players* in the former, *system components* in the latter) taking actions, simultaneously or in turns, on a common set of states ('playground') and thus effecting transitions between these states, modelling an abstract multi-player game in the former framework and respectively an *open system* involving an environment and possibly several competing devices in the latter one. Thus, the *game-theoretic* aspect is dominant in both situations. Furthermore, in both frameworks the agents pursue certain goals with their actions and in that pursuit they can form *coalitions*. The goals of the different coalitions are usually competing, so they typically exhibit adversary behaviours with respect to each other. In both enterprises the objective is to develop formal tools for reasoning about such coalitions of agents and their ability to achieve specified outcomes in these action games.

Following [Pauly, 2000b] in this paper we introduce briefly (multi-player) strategic game frames and models and show that specific classes of them are equivalent to some important types of alternating transition systems. Then we introduce the coalition logics from [Pauly, 2000b], [Pauly, 2000c], and [Pauly, 2000b], show how they embed into fragments of the alternating-time temporal logic **ATL**from [Alur et al, 97], and provide axiomatic systems for these fragments and for the full **ATL.** The paper ends with concluding remarks suggesting how ideas, techniques and results from each of these frameworks could be successfully applied to the other. In order to make the paper self-contained we have included all important definitions from [Pauly, 2000b] and [Alur et al, 97].

A general remark is in order here. The concept of coalitional game traditionally considered in game theory (cf. [Osborne and Rubinstein, 1994]), where every possible coalition is assigned a real number (its *worth*), differs somewhat from the one considered here. In this study we are rather concerned with *qualitative* aspects of game *structures* (frames and models) rather than with *quantitative* analysis of specific games. It should be clear, however, that these two approaches are in agreement and can be easily put together. Indeed, the intermediate link between them is the notion of (qualitative) *effectivity function*, cf. [Pauly, 2000b] where a precise set-theoretic characterization is provided of those abstract effectivity functions (the '*playable*' ones) which are effected by some strategic games. Accordingly, the notion of effectivity function naturally transfers over alternating transition systems, thus providing a framework for purely game-theoretic treatment of alternating temporal logics (which we use for their complete axiomatizations).

2 Multi-player game models and alternating transition systems

2.1 Multi-player strategic game models

Game frames, introduced in [Pauly, 2000b], represent multi-player strategic games where sets of players can form coalitions in attempts to achieve desirable outcomes.

Game frames are based on the notion of a **strategic game form:** a tuple $\langle \mathbf{N}, \{\Sigma_\mathbf{i} | \mathbf{i} \in \mathbf{N}\}, o, \mathbf{S} \rangle$ consisting of:

- a (non-empty) set of **players** $\mathbf{N}$,
- a family of sets of **actions (**or **strategies)** $\Sigma_\mathbf{i}$ for each player $\mathbf{i} \in \mathbf{N}$.
- a set of **states** $\mathbf{S}$,
- an **outcome function** $o : \prod_{\mathbf{i} \in \mathbf{N}} \Sigma_\mathbf{i} \to \mathbf{S}$ which associates with every *strategy profile* (tuple of strategies, one for each of the players) an outcome state in $\mathbf{S}$.

Following [Pauly, 2000b], for every $\mathbf{A} \subseteq \mathbf{N}$ by $\sigma_\mathbf{A}$ we will be denoting a tuple of strategies $\{\sigma_\mathbf{i}\}_{\mathbf{i} \in \mathbf{A}}$ and will be writing $o(\sigma_\mathbf{A}, \sigma_{\mathbf{N}-\mathbf{A}})$ with the presumed meaning. The set of all strategic game forms for a set of players $\mathbf{N}$ over a set of states $\mathbf{S}$ will be denoted by $\Gamma^\mathbf{N}_\mathbf{S}$.

A **multi-player game frame (MGF) for a set of players N** is a pair $(\mathbf{S}, \gamma)$ where $\mathbf{S}$ is a non-empty set of states and $\gamma : \mathbf{S} \to \Gamma^\mathbf{N}_\mathbf{S}$ is a mapping associating a strategic game form with each state in $\mathbf{S}$.

A **multi-player game model (MGM) for a set of players N over a set of propositions** Π is a triple $\mathbf{M} = (\mathbf{S}, \gamma, v)$ where $(\mathbf{S}, \gamma)$ is a multi-player game frame and $v : \mathbf{S} \to \mathbf{2}^\Pi$ is a **valuation** labelling each state from $\mathbf{S}$ with the set of propositions that are true at that state.

The question when two MGMs should be considered 'the same' is a non-trivial one[1]. For the purposes of this paper we will adopt the finest reasonable notion of equivalence between

[1] See the paper "When are two games the same" in [van Benthem, 2000].

MGMs, namely *isomorphism*, i.e. when one can be obtained from the other by means of renaming players and states.

Definition 1 *([Pauly, 2000b]) A strategic game form $\langle \mathbf{N}, \{\Sigma_\mathbf{i} | \mathbf{i} \in \mathbf{N}\}, o, \mathbf{S}\rangle$ is a $\mathbf{d}-$**dictatorship** if there is a player $\mathbf{d} \in \mathbf{N}$ who determines the outcome state of the game, i.e.*

$$\forall \sigma_\mathbf{d} \in \Sigma_\mathbf{d} \exists s \in \mathbf{S} \forall \sigma_{\mathbf{N}-\{\mathbf{d}\}} o(\sigma_\mathbf{d}, \sigma_{\mathbf{N}-\{\mathbf{d}\}}) = s.$$

*A MGM $(\mathbf{S}, \gamma, v)$ is **turn-based**[2] if every $\gamma(s)$ is a dictatorship.*

Definition 2 *A strategic game form $\langle \mathbf{N}, \{\Sigma_\mathbf{i} | \mathbf{i} \in \mathbf{N}\}, o, \mathbf{S}\rangle$ is **injective** if o is injective, i.e. assigns different outcome states to different tuples of strategies.*

*A MGM $(\mathbf{S}, \gamma, v)$ is **injective** if every $\gamma(s)$ is injective.*

2.2 Alternating transition systems

Alternating transition systems, introduced in [Alur et al, 97] building on the concept of *alternation* developed in [Chandra et al, 81], formalize systems of transition effected by collective actions of all agents involved. In the particular case of one agent (the *system*) alternating transition systems are reduced to ordinary transition systems, and the alternating temporal logics, which will be introduced further, associated with one-agent systems are the well known *branching time temporal logics of computations* (see [Emerson, 90]) which formalize *closed systems* having no interaction with other systems, agents, or the environment, and with behaviour determined solely by the states. In the case of two agents the components of an alternating transition system can be thought as a system and an (adversary) environment the behaviour of which is not under the system's control.

An **alternating transition system** (ATS) is defined as in [Alur et al, 97] (with minor notational changes here) as a 5-tuple $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta\rangle$ where:

- Π is a set of (atomic) **propositions,**
- $\mathbf{A}$ is a set of **agents,**
- $\mathbf{Q}$ is a set of **states**,
- $\pi : \mathbf{Q} \to \mathbf{2}^\Pi$ is a **valuation** labelling each state with the set of propositions that are true at that state,
- $\delta : \mathbf{Q} \times \mathbf{A} \to \mathbf{2}^{\mathbf{2}^\mathbf{Q}}$ is a **transition function** mapping a pair (state, agent) to a non-empty family of choices of possible next states. The idea is that at state q an agent $\mathbf{i}$ chooses a set $Q_\mathbf{i} \in \delta(q, \mathbf{i})$ thus forcing the outcome state to be from $Q_\mathbf{i}$. Thus, the choice of the successor state of q is in the intersection of all $Q_\mathbf{i}$ for $\mathbf{i} \in \mathbf{A}$ and so it

[2] In [Pauly, 2000b] these game frames are called *dictatorial,* but we disagree with that term. Indeed, at every local step in such game one player determines the move, but these players can be different for the different moves. Still, we admit that the term 'turn-based' is somewhat restrictive, too. For instance in many collective ball games such as soccer, basketball, etc., at (almost) every moment of time the transition is determined by one player, viz. the one who currently holds the ball, yet they are not turn-based. Still, we will use this term until a better one comes up.

reflects the mutual will of all agents. It is required that for every tuple of choices $\{Q_{\mathbf{i}} | \mathbf{i} \in \mathbf{A}\}$ of all agents the next state is determined, i.e. the intersection $\bigcap_{\mathbf{i}\in\mathbf{A}} Q_{\mathbf{i}}$ is a singleton.

A state s in an ATS $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta \rangle$ is a **successor** of the state q if whenever the system is in a state q the agents can cooperate so that the next state is s, i.e. there are choice sets $Q_{\mathbf{i}} \in \delta(q, \mathbf{i})$, for each $\mathbf{i} \in \mathbf{A}$ such that $\bigcap_{\mathbf{i}\in\mathbf{A}} Q_{\mathbf{i}} = \{s\}$. The set of successors of the state q will be denoted by Q_q^{suc}.

Definition 3 *An ATS* $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta \rangle$ *is* ***tight*** *if for every* $q \in \mathbf{Q}$, $\mathbf{i} \in \mathbf{A}$, *and* $Q \in \delta(q, \mathbf{i})$, $Q \subseteq Q_q^{suc}$.

Remark 1 *Every ATS can be 'tightened' by removing from every* $Q \in \delta(q, \mathbf{i})$ *all states which can never be realized as successors in a transition from* q*. As with MGMs, equivalence of ATSs is a non-trivial matter, but every reasonably general criterion should accept such tightening as equivalent to the original ATS.*

We consider two particular types of ATS, introduced in [Alur et al, 97]: *turn-based synchronous* and *lock-step synchronous*. The intuition behind the former type is that every transition is determined by a *single* agent, but generally different for the different transitions, i.e. the agents can take turns in determining the successive transitions. The intuition behind the latter type is rather opposite: the agents act independently and each of them can only determine its 'local' component of the next state but has no control on the choice of the other agents' components of that state, so the successor state is the resultant of their independent, yet simultaneous actions. Here we somewhat generalize the definitions from [Alur et al, 97], while preserving the underlying ideas.

Definition 4

- An ATS $\mathbf{T} = \langle \Pi, \mathbf{A}, Q, \pi, \delta \rangle$ is **turn-based synchronous** if for every state q there is an agent $\mathbf{i}_q$ such that $\delta(q, \mathbf{i}_q)$ consists of singletons, while for every $\mathbf{j} \neq \mathbf{i}_q$ and $Q \in \delta(q, \mathbf{j})$, $\bigcup \delta(q, \mathbf{i}_q) \subseteq Q$, i.e. $\mathbf{i}_q$ 'decides' which is the next state[3].

- An ATS $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta \rangle$ is **lock-step synchronous** if the set of successor states Q_q^{suc} of every state q can be labelled with all tuples from some Cartesian product[4] $\prod_{\mathbf{i}\in\mathbf{A}} Q_{\mathbf{i}}$ so that for every state q and an agent $\mathbf{i}$ all choice sets from $\delta(q, \mathbf{i})$ are 'hyperplanes' in Q_q^{suc} i.e. sets of the form $\{s_{\mathbf{i}}\} \times \prod_{\mathbf{j}\in\mathbf{A}-\{\mathbf{i}\}} Q_{\mathbf{j}}$, where $s_{\mathbf{i}} \in Q_{\mathbf{i}}$.

  Note that every lock-step synchronous ATS is tight.

[3] We note that this definition can be relaxed even further by allowing the sets in $\delta(q, \mathbf{i}_q)$ to contain also any non-successors. However, we will be mainly interested in tight ATSs where this relazation is vacuous. Also, note that in tight turn-based synchronous ATSs $\delta(q, \mathbf{j}) = \{\bigcup \delta(q, \mathbf{i}_q)\}$ for every $\mathbf{j} \neq \mathbf{i}_q$.

[4] The definition in [Alur et al, 97] requires the *whole* state space Q to be a Cartesian prodict of the 'local' state spaces for each agent. We find that requirement unnecessarily strong.

2.3 From alternating transition systems to multi-player game models

First, every ATS $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta \rangle$ determines an MGM $\mathbf{M^T} = (\mathbf{Q}, \gamma^{\mathbf{T}}, \pi)$ for the set of players $\mathbf{A}$ over the set of states $\mathbf{Q}$, where for each $q \in \mathbf{Q}$ the strategic game form $\gamma^{\mathbf{T}}(q) = \left\langle \mathbf{A}, \left\{ \Sigma_{\mathbf{i}}^{q} | \mathbf{i} \in \mathbf{A} \right\}, o_q, \mathbf{Q} \right\rangle$ is defined as follows, where $\mathbf{A} = \{\mathbf{1}, ..., \mathbf{n}\}$

- $\Sigma_{\mathbf{i}}^{q} = \delta(q, \mathbf{i})$,
- $o_q(Q_{\mathbf{1}}, ..., Q_{\mathbf{n}}) = s$ where $\bigcap_{\mathbf{i} \in \mathbf{A}} Q_{\mathbf{i}} = \{s\}$.

The models $\mathbf{M^T}$ defined as above share a specific property which will be defined below. First, we need an auxiliary technical notion: a **fusion** of n-tuples $(a_1, ..., a_n)$ and $(b_1, ..., b_n)$ is any n-tuple $(c_1, ..., c_n)$ where $c_i \in \{a_i, b_i\}$, $i = 1, ..., n$. The following is easy to check.

Proposition 5 *For any strategic game form* $\langle \mathbf{N}, \{\Sigma_{\mathbf{i}} | \mathbf{i} \in \mathbf{N}\}, o, \mathbf{S} \rangle$, *where* $\mathbf{N} = \{\mathbf{1}, ..., \mathbf{n}\}$ *the following two properties of the outcome function* $o : \prod_{\mathbf{i} \in \mathbf{N}} \Sigma_{\mathbf{i}} \to \mathbf{S}$ *are equivalent:*

(i) *If* $o(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}) = o(\tau_{\mathbf{1}}, ..., \tau_{\mathbf{n}}) = s$ *then* $o(\varsigma_{\mathbf{1}}, ..., \varsigma_{\mathbf{n}}) = s$ *for every fusion* $(\varsigma_{\mathbf{1}}, ..., \varsigma_{\mathbf{n}})$ *of* $(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}})$ *and* $(\tau_{\mathbf{1}}, ..., \tau_{\mathbf{n}})$.

(ii) *For every* $s \in \mathbf{S}$, $o^{-1}(s) = \prod_{\mathbf{i} \in \mathbf{N}} \Delta_{\mathbf{i}}$ *for some* $\Delta_{\mathbf{i}} \subseteq \Sigma_{\mathbf{i}}$, $\mathbf{i} = \mathbf{1}, ..., \mathbf{n}$. ⊣

Definition 6 *A strategic game form* $\langle \mathbf{N}, \{\Sigma_{\mathbf{i}} | \mathbf{i} \in \mathbf{N}\}, o, \mathbf{S} \rangle$, *is* ***convex*** *if the outcome function o satisfies (any of) the two equivalent properties above. A multi-player game model* $\mathbf{M} = (\mathbf{S}, \gamma, v)$ *is* ***convex*** *if* $\gamma(q)$ *is convex for every* $q \in \mathbf{Q}$.

Proposition 7 *For every ATS* $\mathbf{T}$ *the game model* $\mathbf{M^T}$ *is convex.*

Proof: Let $\mathbf{M^T}$ be defined as above. If $o_q(Q_{\mathbf{11}}, ..., Q_{\mathbf{1n}}) = o_q(Q_{\mathbf{21}}, ..., Q_{\mathbf{2n}}) = s$ then $s \in Q_{\mathbf{ji}}$ for each $\mathbf{j} = 1, 2$, $\mathbf{i} = \mathbf{1}, ..., \mathbf{n}$, therefore $\bigcap_{\mathbf{i} \in \mathbf{A}} Q_{\mathbf{j_i i}} = \{s\}$ for any fusion $Q_{\mathbf{j_1 1}}, ..., Q_{\mathbf{j_n n}}$ of $Q_{\mathbf{11}}, ..., Q_{\mathbf{1n}}$ and $Q_{\mathbf{21}}, ..., Q_{\mathbf{2n}}$. ⊣

Remark 2 1. *Pauly has pointed out that the convexity condition is known in game theory under the name of 'rectangularity' and rectangular strategic game forms which are 'tight' in sense that their* α*- and* β*- effectivity functions coincide are characterized in [Abdou, 98] as the normal forms of extensive games with unique outcomes.*

2. *We find the convexity condition which ATSs impose too strong and unjustified in many situations. For instance, consider the following variation of the 'Chicken' game: two cars running against each other on a country road and each of the drivers, seeing the other car, can take any of the actions:* "drive straight", "swerve to the left" *and* "swerve to the right". *Each of the combined actions for the two drivers:* (drive straight, swerve to the left) *and* (swerve to the right, drive straight) *leads to a non-collision outcome, while each of their fusions* (drive straight, drive straight) *and* (swerve to the left, swerve to the right) *leads to a collision. Likewise, in the*

paradigmatic in epistemic reasoning "Simultaneous Byzantine Agreement" any non-coordinated one-sided attack leads to the same outcome – defeat, while the coordinated attack of both armies, which is a fusion of these, leads to a victory. Thus, the definition of outcome function in coalition games is preferable as it renders more general game-theoretic semantics.

2.4 From convex multi-player game models to alternating transition systems and back

Now, with every convex MGM $\mathbf{M} = (\mathbf{S}, \gamma, v)$ for a set of players $\mathbf{N} = \{\mathbf{1}, ..., \mathbf{n}\}$ over a set of propositions Π, where $\gamma(q) = \left\langle \mathbf{N}, \left\{\Sigma_{\mathbf{i}}^{q} | \mathbf{i} \in \mathbf{N}\right\}, o_q, \mathbf{S}\right\rangle$, we associate an ATS $\mathbf{T}^{\mathbf{M}} = \left\langle \Pi, \mathbf{N}, \mathbf{S}, v, \delta^{\mathbf{M}}\right\rangle$ with a transition function $\delta^{\mathbf{M}}$ defined by

$$\delta^{\mathbf{M}}(q, \mathbf{i}) = \left\{ Q_{\sigma_{\mathbf{i}}} = \left\{ o_q(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}) | \sigma_{\mathbf{j}} \in \Sigma_{\mathbf{j}}^{q}, \mathbf{j} \neq \mathbf{i} \right\} | \sigma_{\mathbf{i}} \in \Sigma_{\mathbf{i}}^{q} \right\}.$$

For purely technical reasons we will regard these $\delta^{\mathbf{M}}(q, \mathbf{i})$ as *indexed families* i.e. even if some $Q_{\sigma_{\mathbf{1}}}$ and $Q_{\sigma_{\mathbf{2}}}$ are set-theoretically equal, they will be considered different as long as $\sigma_{\mathbf{1}} \neq \sigma_{\mathbf{2}}$.

By convexity of $\gamma(q)$ it is easy to verify that $\bigcap_{\mathbf{i} \in \mathbf{N}} Q_{\sigma_{\mathbf{i}}} = \{o_q(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}})\}$ for every tuple $(Q_{\sigma_{\mathbf{1}}}, ..., Q_{\sigma_{\mathbf{n}}})$, where $Q_{\sigma_{\mathbf{i}}} \in \delta^{\mathbf{M}}(q, \mathbf{i})$ for $\mathbf{i} = \mathbf{1}, ..., \mathbf{n}$. Furthermore, the following holds obviously.

Proposition 8 *For every convex MGM* $\mathbf{M}$ *the ATS* $\mathbf{T}^{\mathbf{M}}$ *is tight.*

2.5 Equivalence between ATSs and convex MGMs

We have defined constructions converting ATSs into convex MGMs and vice versa. Now we will show that these constructions are mutually inverse, thus proving equivalence between these two types of structures.

Proposition 9

1. *Every tight ATS* $\mathbf{T}$ *is isomorphic to* $\mathbf{T}^{\mathbf{M}^{\mathbf{T}}}$.
2. *Every convex MGM* $\mathbf{M}$ *is isomorphic to* $\mathbf{M}^{\mathbf{T}^{\mathbf{M}}}$.

Proof sketch:

1. It suffices to see that $\delta^{\mathbf{M}^{\mathbf{T}}}(q, \mathbf{i}) = \delta(q, \mathbf{i})$ for every $q \in \mathbf{Q}$ and $\mathbf{i} \in \mathbf{A}$, which is straightforward from the definitions and from the tightness of $\mathbf{T}$. $\dashv$

2. Let $\mathbf{M} = (\mathbf{S}, \gamma, v)$ be a convex MGM and $\gamma(q) = \left\langle \mathbf{N}, \left\{\Sigma_{\mathbf{i}}^{q} | \mathbf{i} \in \mathbf{N}\right\}, o_q, \mathbf{S}\right\rangle$ for $q \in \mathbf{Q}$. For every $\sigma \in \Sigma_{\mathbf{i}}^{q}$ we identify σ with Q_σ defined as above. We have to show that the outcome functions o_q in $\mathbf{M}$ and $\mathbf{o}_q$ in $\mathbf{M}^{\mathbf{T}^{\mathbf{M}}}$ agree under that identification. Indeed, $\mathbf{o}_q(Q_{\sigma_{\mathbf{1}}}, ..., Q_{\sigma_{\mathbf{n}}}) = s$ iff $\bigcap_{\mathbf{i} \in \mathbf{A}} Q_{\sigma_{\mathbf{i}}} = \{s\}$ iff $o_q(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}) = s$. $\dashv$

Proposition 10

1. *Every turn-based game model is convex.*
2. *For every turn-based synchronous ATS* $\mathbf{T}$ *the game model is turn-based. Conversely, if* $\mathbf{M}^{\mathbf{T}}$ *is turn-based for some tight ATS* $\mathbf{T}$*, then* $\mathbf{T}$ *is turn-based synchronous.*
3. *For every convex MGM* $\mathbf{M}$*, the ATS* $\mathbf{T}^{\mathbf{M}}$ *is turn-based synchronous iff* $\mathbf{M}$ *is turn-based.*

Proof sketch:

(1) Let $\mathbf{M} = (\mathbf{S}, \gamma, v)$ be a turn-based MGM for a set of players $\mathbf{N}$, $q \in \mathbf{S}$, and let $\mathbf{d} \in \mathbf{N}$ be the dictator for $\gamma(q)$. Then for every $s \in \mathbf{S}$, $o_q^{-1}(s) = \prod_{\mathbf{i} \in \mathbf{N}} \Delta_{\mathbf{i}}$ where $\Delta_{\mathbf{d}} = \{\sigma_{\mathbf{d}} \in \Sigma_{\mathbf{d}}^q | o_q(..., \sigma_{\mathbf{d}}, ...) = s\}$ and $\Delta_{\mathbf{i}} = \Sigma_{\mathbf{i}}^q$ for all $\mathbf{i} \neq \mathbf{d}$.

(2) and (3) are straightforward. $\dashv$

Proposition 11

1. *Every injective game model is convex.*
2. *For every ATS* $\mathbf{T}$ *the game model* $\mathbf{M}^{\mathbf{T}}$ *is injective iff* $\mathbf{T}$ *is lock-step synchronous.*
3. *For every convex MGM* $\mathbf{M}$*, the ATS* $\mathbf{T}^{\mathbf{M}}$ *is lock-step synchronous iff* $\mathbf{M}$ *is injective.*

Proof sketch:

(1) is trivial.

(2) Let $\mathbf{T}$ be lock-step synchronous and $o_q(\mathbf{Q_1}, ..., \mathbf{Q_n}) = \langle s_{\mathbf{1}}, ..., s_{\mathbf{n}} \rangle$ for some $\mathbf{Q_i} \in \delta(q, \mathbf{i})$, $\mathbf{i} = \mathbf{1}, ..., \mathbf{n}$. Then $\mathbf{Q_i} = \{s_{\mathbf{i}}\} \times \prod_{\mathbf{j} \in \mathbf{A} - \{\mathbf{i}\}} Q_{\mathbf{j}}$, where $Q_q^{suc} = \prod_{\mathbf{i} \in \mathbf{A}} Q_{\mathbf{i}}$, whence the injectivity of $\mathbf{M}^{\mathbf{T}}$. Conversely, if $\mathbf{M}^{\mathbf{T}}$ is injective then every state $s \in Q_q^{suc}$ can be labelled with the unique tuple $\langle \mathbf{Q_1}, ..., \mathbf{Q_n} \rangle$ such that $o_q(\mathbf{Q_1}, ..., \mathbf{Q_n}) = s$, i. e. Q_q^{suc} is represented by $\prod_{\mathbf{i} \in \mathbf{A}} \delta(q, \mathbf{i})$, and every $\mathbf{Q_i} \in \delta(q, \mathbf{i})$ can be identified with $\{\mathbf{Q_i}\} \times \prod_{\mathbf{j} \in \mathbf{A} - \{\mathbf{i}\}} \delta(q, \mathbf{j})$.

(3) If $\mathbf{M}$ is injective then Q_q^{suc} can be labelled by $\prod_{\mathbf{i} \in \mathbf{N}} \Sigma_{\mathbf{i}}^q$ where every $Q_{\sigma_{\mathbf{i}}} \in \delta(q, \mathbf{i})$ is identified with $\{\sigma_{\mathbf{i}}\} \times \prod_{\mathbf{j} \in \mathbf{A} - \{\mathbf{i}\}} \delta(q, \mathbf{j})$. Conversely, if $\mathbf{T}^{\mathbf{M}}$ is lock-step synchronous then every two different $Q_{\sigma_{\mathbf{i}1}}$ and $Q_{\sigma_{\mathbf{i}2}}$ from $\delta(q, \mathbf{i})$ must be disjoint, whence the injectivity of $\mathbf{M}$. $\dashv$

3 Coalition logics and alternating-time temporal logics

3.1 Coalition logic

Coalition logic (CL), introduced in [Pauly, 2000b], formalizes reasoning about powers of coalitions to force outcomes in strategic games. It extends the classical propositional logic with a family of (non-normal) modalities $\{[\mathbf{C}]\,|\mathbf{C} \subseteq \mathbf{N}\}$ where $\mathbf{N}$ is a fixed set of players. Intuitively, $[\mathbf{C}]\varphi$ means that *the coalition* $\mathbf{C}$ *can enforce an outcome state satisfying* φ.

Thus, the formulae of **CL** are defined recursively by

$$\varphi := p|\neg\varphi|\varphi_1 \vee \varphi_2|[\mathbf{C}]\varphi.$$

The semantics of **CL** can be given in terms of **truth at a state of an MGM** $\mathbf{M} = (\mathbf{S}, \gamma, v)$ via the clauses:

- $\mathbf{M}, q \models p$ iff $p \in v(q)$ for atomic propositions p;
- $\mathbf{M}, q \models [\mathbf{C}]\varphi$ iff there is a strategy profile $\sigma_{\mathbf{C}}$ such that for every strategy profile $\sigma_{\mathbf{N-C}}$, $\mathbf{M}, o_q(\sigma_{\mathbf{C}}, \sigma_{\mathbf{N-C}}) \models \varphi$.

Proposition 12 *For every MGM* $\mathbf{M} = (\mathbf{S}, \gamma, v)$ *there is an injective (and hence convex) MGM* $\mathbf{M}' = (\mathbf{S}', \gamma', v')$ *which satisfies the same formulas of* ***CL.***

Proof: For every $\gamma(q) = \left\langle \mathbf{N}, \left\{\Sigma^q_{\mathbf{i}} | \mathbf{i} \in \mathbf{N}\right\}, o_q, \mathbf{S}\right\rangle$ we define $\mathbf{S}_q = \prod_{\mathbf{i}\in\mathbf{N}} \Sigma^q_{\mathbf{i}}$ and let $\mathbf{S}' = \mathbf{S} \cup \bigcup_{q\in\mathbf{S}} \mathbf{S}_q$. Now we define γ' as follows:

- For $q \in \mathbf{S}$, $\gamma'(q) = \left\langle \mathbf{N}, \left\{\Sigma^q_{\mathbf{i}} | \mathbf{i} \in \mathbf{N}\right\}, O^q, \mathbf{S}_q\right\rangle$ where $O^q(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}) = \langle\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}\rangle$
- For $\sigma = \langle\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}\rangle \in \mathbf{S}_q$, and $s = o_q(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}})$, $\gamma'(\sigma) = \gamma(s)$.

Finally, $v'(q) = v(q)$ for $q \in \mathbf{S}$ and $v'(\langle\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}\rangle) = v(o_q(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}))$ for $\langle\sigma_{\ \mathbf{1}}, ..., \sigma_{\mathbf{n}}\rangle \in \mathbf{S}_q$.

The model $\mathbf{M}'$ is injective and for every **CL-** formula φ,

$\mathbf{M}', q \models \varphi$ iff $\mathbf{M}, q \models \varphi$, and

$\mathbf{M}', \langle\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}\rangle \models \varphi$ iff $\mathbf{M}, o_q(\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}) \models \varphi$ for $\langle\sigma_{\mathbf{1}}, ..., \sigma_{\mathbf{n}}\rangle \in \mathbf{S}_q$. $\dashv$

Thus, the restriction of the semantics of **CL** to the class of injective (and hence to convex, as well) MGMs does not introduce new validities.

3.2 Effectivity functions and coalition game models as alternative semantics for coalition logics.

As mentioned earlier, game theory usually measures the powers of coalitions *quantitatively* in terms of*utility/payoff functions* and characterizes the possible outcomes in terms of*payoff profiles.* That approach can be easily transformed into a *qualitative* one, where the payoff profiles are encoded in the outcome states themselves and each coalition is assigned a *preference order* on these outcome states, depending on its expected payoffs associated with them. Eventually, the powers of coalitions can be measured in terms of the *sets of outcome states* in which a coalition can force the actual outcome of the game (i.e. sets for which it is *effective*) in an attempt to maximize its payoff. Thus we arrive at the following notion.

Definition 13 *(Cf. [Pauly, 2000b]) A (local)* ***effectivity function*** *of a strategic game form* $\langle \mathbf{N}, \{\Sigma_\mathbf{i} | \mathbf{i} \in \mathbf{N}\}, o, \mathbf{S}\rangle$ *is a mapping* $E : \mathcal{P}(\mathbf{N}) \to \mathcal{P}(\mathcal{P}(\mathbf{S}))$ *associating with each set of players the family of outcome sets for which their coalition is effective.*

Not every abstract effectivity function corresponds to a real strategic game form. Those which do are called in [Pauly, 2000b] *playable* and characterized there by means of the following simple set-theoretic conditions:

- **outcome-monotone:** if $X \subseteq Y \subseteq \mathbf{S}$ and $X \in E(\mathbf{C})$ then $Y \in E(\mathbf{C})$;
- for every $\mathbf{C} \subseteq \mathbf{N}$, $\emptyset \notin E(\mathbf{C})$;
- for every $\mathbf{C} \subseteq \mathbf{N}$, $\mathbf{S} \in E(\mathbf{C})$;
- **N-maximal:** for all $X \subseteq \mathbf{S}$, if $\mathbf{S} \setminus \mathbf{X} \notin E(\emptyset)$ then $X \in E(\mathbf{N})$;
- **superadditive:** for all $\mathbf{C}_1, \mathbf{C}_2 \subseteq \mathbf{N}$ and $X_1, X_2 \subseteq \mathbf{S}$, if $\mathbf{C}_1 \cap \mathbf{C}_2 = \varnothing$, $X_1 \in E(\mathbf{C}_1)$, and $X_2 \in E(\mathbf{C}_2)$ then $X_1 \cap X_2 \in E(\mathbf{C}_1 \cup \mathbf{C}_2)$.

It has been shown in [Pauly, 2000b] that every playable effectivity function also satisfies the conditions:

- **coalition-monotonicity:** if $\mathbf{C}_1 \subseteq \mathbf{C}_2$ then $E(\mathbf{C}_1) \subseteq E(\mathbf{C}_2)$;
- **regularity:** for all $X \subseteq \mathbf{S}$ and $\mathbf{C} \subseteq \mathbf{N}$, if $X \in E(\mathbf{C})$ then $\mathbf{S} \setminus \mathbf{X} \notin E(\mathbf{N} \setminus \mathbf{C})$;

Thus, instead of using MGMs, the semantics of **CL** can be defined in terms of simpler models as follows.

Definition 14 *A* ***coalition effectivity frame*** *is a triple* $\mathcal{F} = \langle \mathbf{N}, \mathbf{S}, \mathbf{E}\rangle$ *where* $\mathbf{N}$ *is a set of players,* $\mathbf{S}$ *is a non-empty set of states and* $\mathbf{E} : \mathbf{S} \to (\mathcal{P}(\mathbf{N}) \to \mathcal{P}(\mathcal{P}(\mathbf{S})))$ *is a mapping which associates a playable effectivity function with each state. We shall write* $\mathbf{E}_s(\mathbf{C})$ *instead of* $\mathbf{E}(s)(\mathbf{C})$.

A ***coalition effectivity model (CEM)*** *is a pair* $\mathcal{M} = \langle \mathcal{F}, V\rangle$ *where* $\mathcal{F}$ *is a coalition game frame and* V *is a valuation of the atomic propositions of the language of* ***CL*** *in the states of* $\mathcal{F}$.

The semantics of **CL** can now be defined in terms of truth of a formula at a state of a coalition game model, via the clause:

$$\mathcal{M}, s \models [\mathbf{C}]\varphi \text{ iff } \{q \in \mathcal{M} | \mathcal{M}, s \models \varphi\} \in \mathbf{E}_s(\mathbf{C}).$$

Every MGM $\mathbf{M} = (\mathbf{S}, \gamma, v)$ for the set of players $\mathbf{N}$ corresponds to a CEM $E(\mathbf{M}) = \langle \mathcal{F}, v\rangle$ with $\mathcal{F} = \langle \mathbf{N}, \mathbf{S}, \mathbf{E}\rangle$ where for every $s \in \mathbf{S}, X \subseteq \mathbf{S}$, and $\mathbf{C} \subseteq \mathbf{N}$, $X \in \mathbf{E}_s(\mathbf{C})$ iff

$$\exists \sigma_\mathbf{C} \forall \sigma_{\mathbf{N}-\mathbf{C}} \exists q \in X (o(\sigma_\mathbf{C}, \sigma_{\mathbf{N}-\mathbf{C}}) = q)$$

where the strategies above refer to the strategic game form $\gamma(s)$.

Conversely, according to Pauly's characterization, every CEM is isomorphic to some $E(\mathbf{M})$. Thus, the two semantics are equivalent.

3.3 Logics for local and global effectivity of coalitions

While the operators $[\mathbf{C}]\varphi$ can express *local effectivity* properties of coalitions, i.e. their powers to force outcomes in single 'rounds' of the game, Pauly extends in [Pauly, 2001] **CL** to a logic which we shall brand **Global Coalition Logic GCL**), with iterated operators for *global effectivity* $[\mathbf{C}^*]\varphi$, expressing the claim that *the coalition* $\mathbf{C}$ *can achieve the truth of* φ *during the game*, and $[\mathbf{C}^\times]\varphi$, expressing the claim that *the coalition* $\mathbf{C}$ *can maintain the truth of* φ *throughout the entire game.*

In our view, both systems formalize different aspects of reasoning about powers of coalitions: **CL** can be thought as reasoning about *strategic game forms,* where the players' strategies are wrapped into atomic one-step actions, while **GCL** rather deals with *extensive game forms,* representing sequences (possibly infinite) of moves, collectively effected by the players' actions, where the notion of a player's strategy is definable in the usual manner as a function from sequences of states to states.

3.4 Alternating-time temporal logics

Alternating-time temporal logic (ATL), introduced in [Alur et al, 97], formalize reasoning about computations which can be enforced by coalitions of agents, where computations are modelled as transitions in ATSs.

So far, this sounds much like the coalition logic. Indeed, apart from the more expressive language of **ATL** and up to notational and terminological difference, these logical systems are essentially equivalent.

More specifically, ATL extends the computation tree logic CTL with modal operators $\langle\langle \mathbf{C} \rangle\rangle$ and $[[\mathbf{C}]]$ for any set of agents $\mathbf{C}$, meaning respectively

- $\langle\langle \mathbf{C} \rangle\rangle \varphi$: *'The coalition* $\mathbf{C}$*can enforce an outcome satisfying* φ*.'*
- $[[\mathbf{C}]] \varphi$: *'The coalition* $\mathbf{C}$*cannot avoid an outcome satisfying* φ*.'*

Formally, the recursive definition of **ATL** formulas is:

$$\varphi := p|\neg\varphi|\varphi_1 \vee \varphi_2| \langle\langle \mathbf{C} \rangle\rangle X\varphi| \langle\langle \mathbf{C} \rangle\rangle G\varphi| \langle\langle \mathbf{C} \rangle\rangle \varphi_1 \mathcal{U} \varphi_2$$

where the temporal operators X (*nexttime*), G (*always*) and $\mathcal{U}$ (*until*) have the usual meaning. Note that $\langle\langle \mathbf{C} \rangle\rangle F\varphi$ is definable as $\langle\langle \mathbf{C} \rangle\rangle \top\mathcal{U}\varphi$. Also, the operator $[[\mathbf{C}]]$ is definable as the dual of $\langle\langle \mathbf{C} \rangle\rangle$.

The semantics of **ATL** is based on alternating transition systems as follows.

Definition 15 *Let* $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta \rangle$ *be an ATS,* $q \in \mathbf{Q}$*. A state* q' *is an* $\mathbf{i}$ ***-successor of*** q *for* $\mathbf{i} \in \mathbf{A}$ *if* $q' \in Q$ *for some* $Q \in \delta(q, \mathbf{i})$*;* q' *is a* ***successor of*** q *if it is an* $\mathbf{i}$*-successor of* q *for each* $\mathbf{i} \in \mathbf{A}$*. A* q*-**computation** in* $\mathbf{T}$ *is an infinite sequence* $q = q_0, q_1, ...$ *such that* q_{i+1} *is a successor of* q_i *for every* $i \geq 0$.

Definition 16 *Let* $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta \rangle$ *be an ATS. A* ***strategy in*** $\mathbf{T}$ ***for an agent*** $\mathbf{i} \in \mathbf{A}$ *is a mapping* $f_\mathbf{i} : \mathbf{Q}^+ \to \mathbf{2}^\mathbf{Q}$ *which assigns to every non-empty sequence of states* $q_0, ..., q_n$

a choice set $f_{\mathbf{i}}(\langle q_0, ..., q_n\rangle) \in \delta(q_n, \mathbf{i})$*. A* ***strategy profile in*** $\mathbf{T}$ ***for a set of agents*** $\mathbf{C} \subseteq \mathbf{A}$ *is a family of strategies* $F_{\mathbf{C}} = \{f_{\mathbf{i}}\}_{\mathbf{i}\in\mathbf{C}}$*. Given a state* q*, a set of agents* $\mathbf{C} \subseteq \mathbf{A}$ *and a strategy profile* $F_{\mathbf{C}} = \{f_{\mathbf{i}}\}_{\mathbf{i}\in\mathbf{C}}$*, the* ***set*** $out(q, F_{\mathbf{C}})$ ***of outcomes of*** $F_{\mathbf{C}}$ ***from*** q *is the set of all* q*-computations* $q = q_0, q_1, ...$ *such that* $q_{i+1} \in \bigcap_{\mathbf{i}\in\mathbf{C}} f_{\mathbf{i}}(q_i)$ *for every* $i \geq 0$*.*

Now the definition of a **truth of an ATL-formula at a state** q **of an ATS** $\mathbf{T} = \langle \Pi, \mathbf{A}, \mathbf{Q}, \pi, \delta\rangle$ goes through the following clauses, where $\mathbf{C} \subseteq \mathbf{A}$:

$(\mathbf{C}, \mathbf{X})$ $\mathbf{T}, q \models \langle\langle \mathbf{C}\rangle\rangle X\varphi$ iff there exists a strategy profile $F_{\mathbf{C}} = \{f_{\mathbf{i}}\}_{\mathbf{i}\in\mathbf{C}}$ such that for every q-computation $q = q_0, q_1, ... \in out(q, F_{\mathbf{C}})$, $\mathbf{T}, q_1 \models \varphi$.

$(\mathbf{C}, \mathbf{G})$ $\mathbf{T}, q \models \langle\langle \mathbf{C}\rangle\rangle G\varphi$ iff there exists a strategy profile $F_{\mathbf{C}} = \{f_{\mathbf{i}}\}_{\mathbf{i}\in\mathbf{C}}$ such that for every q-computation $q = q_0, q_1, ... \in out(q, F_{\mathbf{C}})$, $\mathbf{T}, q_i \models \varphi$ for every $i \geq 0$.

$(\mathbf{C}, \mathcal{U})$ $\mathbf{T}, q \models \langle\langle \mathbf{C}\rangle\rangle \varphi\mathcal{U}\psi$ iff there exists a strategy profile $F_{\mathbf{C}} = \{f_{\mathbf{i}}\}_{\mathbf{i}\in\mathbf{C}}$ such that for every q-computation $q = q_0, q_1, ... \in out(q, F_{\mathbf{C}})$ there is $i \geq 0$ such that $\mathbf{T}, q_i \models \psi$ and for all j such that $0 \leq j < i$, $\mathbf{T}, q_j \models \varphi$.

As mentioned before, every ATS can be 'tightened' at each state by removing from the choice sets for all agents those states which are not successors of that state. The result of such tightening will not affect the truth of the formulae of **ATL** at that state. Thus:

Proposition 17 *For every ATS* $\mathbf{T}$ *there is a tight ATS* $\mathbf{T}'$ *which satisfies the same formulae of* ***ATL.***

3.5 Embedding CL and GCL into ATL

Note that the clause $(\mathbf{C}, X)$ above can be rephrased as:

[**C**] $\mathbf{T}, q \models \langle\langle \mathbf{C}\rangle\rangle X\varphi$ iff there exist a strategy profile $F_{\mathbf{C}} = \{f_{\mathbf{i}}\}_{\mathbf{i}\in\mathbf{C}}$ such that for every strategy profile $F_{\mathbf{A}-\mathbf{C}} = \{f_{\mathbf{j}}\}_{\mathbf{j}\in\mathbf{A}-\mathbf{C}}$, $\mathbf{T}, s \models \varphi$, where $\{s\} = \bigcap_{\mathbf{i}\in\mathbf{C}} f_{\mathbf{i}}(\langle q\rangle) \cap \bigcap_{\mathbf{j}\in\mathbf{A}-\mathbf{C}} f_{\mathbf{j}}(\langle q\rangle)$,

which is precisely the truth-condition for $[\mathbf{C}]\varphi$ in the coalition logic **CL.**

Thus, **CL** embeds in an obvious way as a simple fragment of **ATL** by translating $[\mathbf{C}]\varphi$ into $\langle\langle \mathbf{C}\rangle\rangle X\varphi$. We denote the resulting fragment by $\mathbf{ATL}_X$. In fact, **ATL** extends **CL** just like **CTL** extends the modal logic **K** over the language with modality $\forall X$.

Furthermore, $[\mathbf{C}^*]\varphi$ translates into **ATL** as $\langle\langle \mathbf{C}\rangle\rangle F\varphi$ while $[\mathbf{C}^\times]\varphi$ translates as $\langle\langle \mathbf{C}\rangle\rangle G\varphi$, hence **GCL** is embeddable into **ATL**, too, as the fragment $\mathbf{ATL}_{XG}$ involving only $\langle\langle \mathbf{C}\rangle\rangle X\varphi$ and $\langle\langle \mathbf{C}\rangle\rangle G\varphi$.

3.6 Axiomatic systems for alternating temporal logics

In [Pauly, 2000b] Pauly gives a complete axiomatization of **CL** with respect to coalition effectivity models. Combining this result with the equivalence of the semantics for **ATL** and **CL** it is easy to show that the axiomatic system for **CL** translates into a complete axiomatization of $\mathbf{ATL}_X$.

Furthermore, that can be extended to axiomatizations of $\mathbf{ATL}_{XF}$, $\mathbf{ATL}_{XG}$, and the full **ATL**, the completeness of which will be proved elsewhere, by adding axioms expressing the facts that $\langle\langle \mathbf{C} \rangle\rangle\, G\varphi$, $\langle\langle \mathbf{C} \rangle\rangle\, F\varphi$, and $\langle\langle \mathbf{C} \rangle\rangle\, \varphi_1 \mathcal{U} \varphi_2$ are certain greatest or least fixed point operators. Note that $\langle\langle \varnothing \rangle\rangle\, G$ expresses the usual temporal operator G.

3.6.1 Axioms for $\mathbf{ATL}_{XG}$:

(CL) The **CL** axioms for $\langle\langle \mathbf{C} \rangle\rangle\, X\varphi$ from [Pauly, 2000b] expressing the characterizing conditions of playable effectivity functions:

(⊥) $\neg \langle\langle \mathbf{C} \rangle\rangle\, X\bot$,

(⊤) $\langle\langle \mathbf{C} \rangle\rangle\, X\top$,

(N) $\neg \langle\langle \varnothing \rangle\rangle\, X\neg\varphi \to \langle\langle \mathbf{A} \rangle\rangle\, X\varphi$,

(M) $\langle\langle \mathbf{C} \rangle\rangle\, X(\varphi \wedge \psi) \to \langle\langle \mathbf{C} \rangle\rangle\, X\varphi$[5],

(S) $\langle\langle \mathbf{C}_1 \rangle\rangle\, X\varphi \wedge \langle\langle \mathbf{C}_2 \rangle\rangle\, X\psi \to \langle\langle \mathbf{C}_1 \cup \mathbf{C}_2 \rangle\rangle\, X(\varphi \wedge \psi)$ for disjoint $\mathbf{C}_1$ and $\mathbf{C}_2$.

Additional axioms for $\langle\langle \mathbf{C} \rangle\rangle\, G$:

(FP$_G$) $\langle\langle \mathbf{C} \rangle\rangle\, G\varphi \leftrightarrow \varphi \wedge \langle\langle \mathbf{C} \rangle\rangle\, X\, \langle\langle \mathbf{C} \rangle\rangle\, G\varphi$,

(GFP$_G$) $\langle\langle \varnothing \rangle\rangle\, G(\theta \to (\varphi \wedge \langle\langle \mathbf{C} \rangle\rangle\, X\theta)) \to \langle\langle \varnothing \rangle\rangle\, G(\theta \to \langle\langle \mathbf{C} \rangle\rangle\, G\varphi))$.

Rules of inference: *Modus Ponens*,

$\langle\langle \mathbf{C} \rangle\rangle\, X$ *-Monotonicity:*

$$\frac{\varphi \to \psi}{\langle\langle \mathbf{C} \rangle\rangle\, X\varphi \to \langle\langle \mathbf{C} \rangle\rangle\, X\psi}$$

and $\langle\langle \mathbf{C} \rangle\rangle\, G$*-Necessitation:*

$$\frac{\varphi}{\langle\langle \mathbf{C} \rangle\rangle\, G\varphi}$$

If an appropriate determinacy condition, equivalent to **C**-maximality for every coalition **C**, is assumed then the operator $\langle\langle \mathbf{C} \rangle\rangle\, F$ is definable in $\mathbf{ATL}_{XG}$ by means of

$$\langle\langle \mathbf{C} \rangle\rangle\, F\varphi = \neg \langle\langle \mathbf{N} - \mathbf{C} \rangle\rangle\, G\neg\varphi.$$

On the other hand, since **N**-maximality is always assumed, $\langle\langle \varnothing \rangle\rangle\, G\varphi$ is equivalent to $\neg \langle\langle \mathbf{N} \rangle\rangle\, F\neg\varphi$, hence $\langle\langle \mathbf{C} \rangle\rangle\, F$ can be axiomatized in $\mathbf{ATL}_{XF}$ with the respective axioms:

(FP$_F$) $\langle\langle \mathbf{C} \rangle\rangle\, F\varphi \leftrightarrow \varphi \vee \langle\langle \mathbf{C} \rangle\rangle\, X\, \langle\langle \mathbf{C} \rangle\rangle\, F\varphi$,

(LFP$_F$) $\langle\langle \varnothing \rangle\rangle\, G((\varphi \vee \langle\langle \mathbf{C} \rangle\rangle\, X\theta) \to \theta) \to \langle\langle \varnothing \rangle\rangle\, G(\langle\langle \mathbf{C} \rangle\rangle\, F\varphi \to \theta)$.

[5]This axiom is redundant, because we adopt here the stronger rule of Monotonicity rather than Equivalence, as in [Pauly, 2000b].

Finally, the axiomatic system for the full **ATL** is obtained from $\mathbf{ATL}_{XG}$ by adding the (stronger) versions for $\mathcal{U}$ of the latter two axioms:

($\mathbf{FP}_{\mathcal{U}}$) $\langle\langle \mathbf{C} \rangle\rangle\, \varphi\mathcal{U}\psi \leftrightarrow \psi \vee \langle\langle \mathbf{C} \rangle\rangle\, X(\varphi \wedge \langle\langle \mathbf{C} \rangle\rangle\, \varphi\mathcal{U}\psi)$,

($\mathbf{LFP}_{\mathcal{U}}$) $\langle\langle \varnothing \rangle\rangle\, G((\psi \vee \langle\langle \mathbf{C} \rangle\rangle\, X(\varphi \wedge \theta)) \rightarrow \theta) \rightarrow \langle\langle \varnothing \rangle\rangle\, G(\langle\langle \mathbf{C} \rangle\rangle\, \varphi\mathcal{U}\psi \rightarrow \theta)$.

4 Concluding remarks

We have done a comparative study of coalition game logics and alternating time logics and have demonstrates their intimate relationship. Still these two enterprises differ in their motivations and agendas. Yet, they can borrow many ideas and results, both technical and conceptual, from the other.

In particular, here are some aspects of ATL which seem worth applying to coalition games:

- Turn-based asynchronous ATSs, introduced in [Alur et al, 97] can offer a formalization of turn-based games by adding a fictitious player, a *scheduler,* the effect of whose actions is to determine the players' turns.
- *Fairness constraints* are natural in (alternating) computations. What is their meaning in coalition games?
- *Alternating refinement relations* (see [Alur et al, 98]) offer the appropriate notion of bisimulation between ATSs and thus can suggest an answer to the question "*When are two coalition games equivalent?*"
- ATSs and ATL with *incomplete information*, introduced in [Alur et al, 97] naturally correspond to coalition games and logics with imperfect information. Also, stronger languages and logics such as $\mathbf{ATL}^*$ and alternating-time μ-calculus, discussed there provide more expressive tools for reasoning about coalition games.
- A number of expressiveness and complexity results, as well as realizability and model-checking methods from [Alur et al, 97] and [Alur et al, 98] can be transferred to coalition games.

Conversely, coalition games and logics, too, offer some ideas, results and agendas to ATL:

- As already discussed, MGMs provide a more general semantics, worth developing in the framework of ATL.
- Furthermore, as demonstrated in [Pauly, 2000b, Pauly, 2000c, Pauly, 2001], *effectivity functions* provide simpler game models and technically handier semantics, essentially based on neighbourhood semantics for non-normal modal logics (see [Parikh, 85] and [Pauly, 2000a]). These can be applied likewise to ATL.
- Fundamental concepts in game theory are *preference relations between outcomes,* and *Nash equilibria.* Their counterparts in alternating transition systems are unexplored yet.

In conclusion, we see the main contribution of the present study as casting the bridge between the two frameworks, intended to trigger a synergetic effect from their mutual influence.

Acknowledgments

I thank Johan van Benthem for sparking my interest in logical aspects of games, Marc Pauly for stimulating discussions on coalition games and valuable remarks on this paper, Moshe Vardi for drawing my attention to [Alur et al, 98], and Philippe Balbiani and an anonimous referee for careful reading and corrections on the text.

References

[Abdou, 98] J. Abdou, Rectangularity and Tightness: A Normal Form Characterization of Perfect Information Extensive Game Forms, Mathematics and Operations Research, vol. 23 (3), 1998.

[Alur et al, 97] R. Alur, T.A. Henzinger, and O. Kupferman, Alternating-time temporal logic, in: **Proceedings of the 38th {IEEE} Symposium on Foundations of Computer Science,** Florida, October 1997.

[Alur et al, 98] R. Alur, T.A. Henzinger, O. Kupferman, and M. Vardi, Alternating Refinement Relations, in: **CONCUR'98**, 1998.

[van Benthem, 2000] J. van Benthem, Logic and Games, ***Lecture notes, ILLC report X-2000-03,*** available at http://www.illc.uva.nl/Publications/Lists/www-allpubl-2000.html.

[Chandra et al, 81] A.K. Chandra, D. Kozen and L. Stockmeyer, Alternation, *Journal of the ACM,* **28(1),** 1981, 114-133.

[Emerson, 90] A. Emerson, Temporal and Modal Logics, in: Leeuwen, J.˜van (ed.), **Handbook of Theoretical Computer Science**, **vol. B: Formal Models and Semantics**, Elsevier, 1990, 995–1072.

[Osborne and Rubinstein, 1994] M. Osborne and A. Rubinstein, **A Course in Game Theory,** MIT press, 1994.

[Pauly, 2000a] M. Pauly, Game Logic for Game Theorists, *CWI report INS-R0017,* Sept. 2000.

[Pauly, 2000b] M. Pauly, A Modal Logic for Coalition Power in Games, *Journal of Logic and Computation,* to appear.

[Pauly, 2000c] M. Pauly, A Logical Framework for Coalitional Effectivity in Dynamic Procedures, Proceedings of the 4th Conference on logic and the Foundations of Game and Decision Theory (LOFT4), Torino, 2000.

[Pauly, 2001] M. Pauly, Logic for Social Software, ***Ph. D. Thesis*** (manuscript), ILLC, Univ. of Amsterdam, 2001.

[Parikh, 85] R. Parikh, The Logic of Games and its Applications, Ann. of Discrete Mathematics, vol 24 (1985), 111-140.