Preplay negotiations by unconditional offers in 2-player strategic games: power and weakness*

Valentin Goranko
Stockholm University

June 3, 2016

Abstract

I consider an extension of strategic form games with a phase of negotiations before the actual play of the game, where players can make series of unilaterally binding offers for incentive payments of utilities to other players after the play of the game, conditional only on the recipients playing the strategy indicated in the offer. The effect of such offer is a transformation of the payoff matrix of the game by accordingly transferring the offered amount of utility from the offering player to the recipient in all outcomes where the indicated strategy is played by the latter.

In this paper we focus on the case when the players can make no assumptions or predictions about the behaviour of their opponents in the preplay negotiations phase, which can terminate at any stage. Consequently, in their negotiation strategy the players are guided by an *Immediate rationality assumption*, and therefore aim at optimising each of their offers. Thus, here we study *locally rational behaviour* of players, who follow step-wise optimal strategies in pursuit of the long-term objective of optimising the outcome of the resulting strategic game. The examples and analysis presented here show that under these assumptions the preplay negotiations can develop in essentially different ways, generally improving the payoffs of both players but not always leading to optimal outcomes.

1 Introduction

The traditional approach to non-cooperative normal form games makes no allowance for possible communication between the players prior to the game. This is one of the inherent reasons why some normal form games, like the Prisoner's Dilemma, have rather unsatisfactory – e.g., strongly Pareto dominated – equilibria solutions. The argument is that the lack of communication between the players leads to the impossibility for them to negotiate before the play of the game with the aim of trying to reach mutually more

*This is a shorter version of the working paper [3] beneficial outcomes in the actual game, while still playing it in a non-cooperative mode. A natural mechanism enabling the possibility of such preplay negotiations has recently been studied in [5] and [6]. It is based on the assumption that players are able to communicate before playing the game and make *unilaterally binding offers to other players*, for payments of explicitly declared amounts of utility, only contingent on the strategies played by the recipients of these offers. More precisely, the assumption proposed and studied in [5] and [6] is that *before the actual game is played, any player X can make a binding offer for payment to any other player Y to pay him, after the game is played, an explicitly declared amount of utility δ if Y plays a strategy s specified in the offer by X*[1]. The structure and outcome of the emerging *preplay negotiations game* between the players exchanging such offers depends crucially on some additional assumptions about the nature of the negotiation process, i.e. whether or not offers can be conditional on the opponents responding with matching offers, as well as whether offers can be withdrawn, whether players' time is valuable, etc. The case of preplay negotiations with conditional offers has been analysed in [5] and [6], where we have shown that assuming valuable time – but not in general – the preplay negotiation phase essentially falls in the framework of Rubinstein's bargaining model [11, 10]. The reader is also referred to [6] for an extensive discussion on related work and comparisons with other approaches, to the full paper [3] for more details and discussion, and to [4] which initiates a study of extensive games with interplay offers.

For further details and discussions of such kinds of scenarios and related approaches to preplay negotiations see e.g. [7], [8], [12], [13], [9], [1], [14].

Building on the framework of preplay offers and negotiations game developed in [5] and [6], here I consider and analyse the case when players can only exchange *unconditional offers* for incentive payments and, moreover, cannot make any assumptions or predictions about the behaviour of their opponents in the preplay negotiations game. In particular, players can expect that the preplay negotiations game can terminate immediately after their next offer, as their opponents may not be willing or able to make any further suitable offers in response. This is a natural and common assumption when players do not know well enough the others' solution concepts, methods for computing the value of the game to be played, degree of patience and value of time, etc., and therefore the others' rational behaviour. Under this assumption, the players are guided in their negotiation strategy by an *immediate rationality* reasoning and therefore always aim at optimising individually each of their offers without expecting any cooperation by their opponents in extending further the negotiation process. Thus, here we study *locally rational behaviour* of players, who follow step-wise optimal strategies in pursuit of the long-term objective of optimising the outcome of the resulting strategic game. The examples and analysis presented here show that under these assumptions the preplay negotiations can develop in essentially different ways, generally improving the payoffs of both players but not always leading to optimal outcomes. In particular, the bargaining power of unconditional offers turns out to be substantially weaker than the bargaining power of players who can also exchange conditional offers, studied in [5] and [6].

[1] Clearly, such offers can only work when the payoffs are with transferrable utilities.

It is important to emphasise, however, that the preplay offers are assumed *only unilaterally binding on the offering players but not on the recipients*, and thus the game remains *non-cooperative in nature.* Still, the examples and results mentioned here clearly indicate that the possibility of making such unilaterally binding incentive offers leads in general to much more efficient and mutually beneficial solutions, thus providing a game-theoretic platform for the emergence of cooperation in an inherently non-cooperative setting.

2 Preliminaries and background

Here we only provide a brief summary of the framework of non-cooperative games with preplay negotiations. For details, see [6].

2.1 Normal form games

We define general N-player normal form game (hereafter abbreviated as NFG), even though this paper will only consider 2-player games, as a tuple $\mathcal{G} = (N, \{\Sigma_i\}_{i\in N}, u)$, where $N = \{1,\ldots,n\}$ a finite set of players, $\{\Sigma_i\}_{i\in N}$ a family of strategies for each player and $u : N \times \prod_{i\in N} \Sigma_i \to \mathbb{R}$ is a **payoff function** assigning to each player a utility for each strategy profile. The game is played by each player i choosing a strategy from Σ_i. The resulting strategy profile σ is the **outcome** of the play and $u_i(\sigma) = u(i,\sigma)$ is the associated payoff for i. An outcome σ is **dominated** by an outcome σ' if $u_i(\sigma) \leq u_i(\sigma')$ for each $i = \{1,\ldots,n\}$ and $u_i(\sigma) < u_i(\sigma')$ for some such i. An outcome is **Pareto optimal** if it is not dominated by any outcome. The **total value** of an outcome σ is the sum of all payoffs $u_1(\sigma)+\ldots+u_n(\sigma)$. An outcome is **maximal** if it has the highest total value amongst all outcomes of the game. Clearly, every maximal outcome is Pareto optimal.

2.2 Preplay offers and induced game transformations

Consider a generic two-player normal form game, with players A and B having respective strategies $A_1, ..., A_i, ...$ and $B_1, ..., B_j, ...$, with outcomes defined by a given payoff matrix. We assume that before the play of that game, each player, say X, can make a *unilaterally binding offer* to the other player, Y, for payment (transfer) of amount of utility $\delta \geq 0$ [2] after the play of the game, conditional on Y playing the strategy s specified in the offer. We will use the following notation for such offer: $X \xrightarrow{\delta/s} Y$. Such offer *does not* create any obligation for the recipient Y and therefore it does not transform the game into a cooperative one, for Y is still free to choose his strategy when the game is actually played. The technical effect of an offer is a transformation of the payoff matrix by accordingly transferring the offered amount of utility from the offering player to the recipient in all outcomes where the indicated strategy is played by the latter. We will call such game transformations **offer-induced transformations**, or just **OI-transformations**. For

[2] The reason to allow vacuous offers with $\delta = 0$ is not only for technical convenience, but also because such offers can be used by players as signalling, to enable coordination in cases where there is more than one equilibrium yielding the same payoff for the other player; see more on that in Section 4.

further details and discussion on these see [6] and for a general mathematical study and characterisation of the OI-transformations of payoff matrices, see [2]. Here we only note that they do not change the total value of any outcome, neither the preferences of the offering player regarding her own strategies. Furthermore, the players can collude to redistribute the payoffs in any chosen outcome in any possible way, by exchanging suitable offers contingent on the strategies generating that outcome. Moreover, they can collude to make any chosen outcome a dominant strategy equilibrium, by exchanging sufficiently high offers to make the strategies generating that outcome strictly dominant.

2.3 Preplay offers and game transformations: two examples

Prisoners' Dilemma 1 Consider a standard version of the Prisoner' s Dilemma (PD) game in Figure 1. The only Nash Equilibrium (NE) of the game on the left is (D, D), which is also the maximin solution and the only outcome surviving elimination of strictly dominant strategies, yielding a payoff of $(1, 1)$.

	C	D
C	$5,5$	$0,6$
D	$6,0$	$1,1$

	C	D
C	$3,7$	$0,6$
D	$4,2$	$1,1$

	C	D
C	$5,5$	$2,4$
D	$4,2$	$1,1$

Figure 1: Prisoner's Dilemma: the initial game and the transformations after the offers

Now, suppose player *Row* makes an offer $Row \xrightarrow{2/C} Column$ to player *Column* to pay her 2 units of utility (hereafter, *utils*) after the game if *Column* plays C. That offer transforms the game by transferring 2 utils from the payoff of *Row* to the payoff of *Column* in every entry of the column where *Column* plays C, as pictured in the middle matrix. In this game player *Row* still has the preference to play D, which strictly dominates C for him, but the dominant strategy for *Column* now is C, and thus the only Nash equilibrium (and the only maximin solution) is (D, C) with payoff $(4, 2)$ – strictly better for both players than the original payoff $(1, 1)$. Thus, even though *Row* will still defect, the offer he has made to *Column* makes it strictly preferable for *Column* to cooperate. Furthermore, *Column* can now realise that she would be even better off if *Row* would cooperate, too, but for that an extra incentive for *Row* is needed. Such incentive can be created e.g. by an offer $Column \xrightarrow{2/C} Row$, which further transforms the game as in the matrix on the right, where the only equilibrium in the game is (C, C) is a maximal outcome, thus solving the game is the best possible way for both players.

Prisoners' Dilemma 2 Consider another version of the Prisoner' s Dilemma game:

	C	D
C	$4,4$	$0,5$
D	$5,0$	$3,3$

The only Nash Equilibrium in this game (and the only maximin solution) is again (D, D), yielding the Pareto dominated payoff of $(3, 3)$. However, as noted in [3], unlike the previous example, here none of the players can make a 'rational' first offer to improve the outcome. Thus, while unconditional preplay offers provide a strong bargaining power, they do not trivialise non-cooperative games by turning them into cooperative games.

2.4 Solution concepts and players' values of normal form games

Solution concepts are discussed in general in [3]. Without committing to a specific solution concept for now, we will assume that the one adopted by the players satisfies the following necessary condition for every outcome in any solution prescribed by that solution concept:

i) *it must survive iterated elimination of strictly dominated strategies (IESDS)*,

ii) *it must yield at least the maximin value of the game for each player*.

iii) *If the game has just one pure strategy Nash equilibrium, then it is the only outcome in the solution.*

We will call such solution concepts **acceptable**. Thus, the weakest acceptable solution concept returns all outcomes that survive IESDS and where each player gets at least their maximin value of the game.

It is necessary for the preplay negotiation phase introduced later that each player has a value of any NFG that can be played. How that value is computed has a limited effect of the general theory and can be left as a separate issue. Here we adopt the conservative, risk-averse approach and assume for every acceptable solution concept $\mathfrak{S}$, game $\mathcal{G}$, and player i, **the value of $\mathcal{G}$ for i relative to the solution concept $\mathfrak{S}$** to be *the minimal payoff that the player gets from an outcome in the $\mathfrak{S}$-solution of the game $\mathcal{G}$.*

2.5 Preplay negotiations games: basic concepts

The concepts and definitions presented in this section are extracted from [6]. Our setting for normal form games with preplay offers begins with a given, 'input' normal form game $\mathcal{G}$ and consists of two phases: The first one is a **preplay negotiation phase**, where players negotiate on how to transform the game $\mathcal{G}$ by making unconditional and irrevocable offers to each other. This phase constitutes an extensive form game, which we call a **preplay negotiation game (PNG)**. The second phase is the **actual play** of the resulting transformed game. Each player's objective in the preplay negotiations is reaching a best for them possible agreement on the OI-transformation of the original game $\mathcal{G}$.

The actual structure and possible outcomes of the preplay negotiation games depend essentially on several important *additional assumptions*, including *conditionality and revocability of offers*, *value of time*, and *the order of making offers.* These options and their effects on the preplay negotiations have been discussed in some detail in [6]. Here we assume that *all offers are unconditional and irrevocable, the time is not valuable for any of the players, and the offers are made in struct alternation.*

An informal definition of PNG under the additional assumptions made above is given in [3] as a special kind of extensive-form bargaining games. For the formal definition in the general case and further details, see [6]. Hereafter we restrict attention to 2-player games, though much of what follows still applies to the general n-player case.

3 Preplay negotiation games with immediate rewards

3.1 The Immediate Rationality Assumption

Hereafter we adopt the following **Immediate Rationality Assumption**, hereafter abbreviated as IRA, essentially stating that *the players assume that the preplay negotiation game can be terminated unilaterally by the opponent at any time.* That happens when the opponent stops making any further offers or simply opts out. Consequently, rational players in such negotiation games have to make sure that every offer they make improves optimally their value of the transformed game, as it may turn out to be the final offer in the preplay negotiations. This assumption is often well justified, e.g., when players have no a priori knowledge about each other's rationality, guiding solution concept for computing their value of the game, and patience.

Under the IRA assumption, whenever the players are to make the next move (offer) in the PNG, they have to calculate an *optimal offer* on the current NFG without taking into account any possible continuations of the preplay negotiation after their moves. Thus, an optimal strategy of a player in such PNG would only prescribe moves that would guarantee, as a minimum, that the resulting transformed NFG has no lesser value for the player making that move than the currently accepted NFG. To put it simple, IRA prescribes to the players to play 'stepwise optimal' strategies in the PNG that result in immediate rewards[3]. Thus, here we have to analyze the rationality of *moves*, rather than long term strategies. Consequently, adopting the IRA assumption makes the analysis somewhat easier. However, as we will see further, it still remains quite intricate, because a player who makes an unconditional offer in fact makes a unilateral concession for the mutual benefit and thereby can put himself in a disadvantaged position by transforming the payoff matrix to the other player's sole benefit. Therefore, generally, players are more interested in receiving, rather than in making, unconditional offers and this affects essentially the preplay negotiations phase.

3.2 Efficient negotiation strategies under IRA

In order to carry out our analysis and to make justified statements about solutions assuming IRA, we first need to clarify the important notions of 'feasibility of moves' and 'efficiency of negotiation strategies'. In this context we will use the term 'efficient' not in its traditional game-theoretic sense, i.e., applied to outcomes, but to the way outcomes are reached.

3.2.1 Types of unconditional offers

We can distinguish 3 types of unconditional offers:

1. **vacuous offers**, of the kind $A \xrightarrow{0/\sigma} B$ for payment of 0. These can be used instead of passing, but also – more importantly – as a kind of *signaling*, i.e., indication that

[3] The IRA-guided reasoning is similar in spirit to the 'greedy algorithms' approach in programming.

A expects B to play σ, for the sake of breaking the symmetry in case of symmetric games with several equivalent optimal equilibria.

2. **ϵ-offers**, of the kind $A \xrightarrow{\epsilon/\sigma} B$ for a small enough $\epsilon > 0$, which we call *rationality threshold*, see further. These offers can be used, similarly, for breaking the symmetry when B has more than one best for him moves which, however, leading to outcomes that yield different payoffs for A. Using such a move, A can make any of these outcomes strictly preferable for B and, thus, can turn a weak equilibrium into a strict one with minimal cost.
3. **effective offers**, of the kind $A \xrightarrow{d/\sigma} B$ for a (large enough) $d > 0$. This is the main type of offers, used to change the recipient's preferences and influence his choice of strategy in the transformed NFG.

It is easy to see that in ideal negotiations none of the players needs to make two consecutive offers, between which the opponent has passed or has made a vacuous offer. Indeed, no player would be better off by making competing offers, contingent on two or more different strategies of the opponent; in fact, such multiple offers would send confusing signals to the opponent. Furthermore, two or more offers by the same player that are contingent on the same strategy of the opponent can be combined into one. So, under our assumptions we can restrict attention to 2-player PNGs that consist of a sequence of strictly alternating offers made in turn by the 2 players until both of them pass or opt out.

3.2.2 Feasible offers and strategies

Following the discussion above, we say that a player's offer is **weakly feasible** if it does not decrease that player's value of the current game in the game transformed by that offer; the offer is **feasible** if it strictly increases that value.

Respectively, we call a strategy of a player in a PNG **feasible** if it only involves making (at least weakly) feasible offers and eventually opts out from the negotiations game. Here we argue that, assuming IRA, in order for a player's strategy in the preplay negotiation phase to be part of a rational solution, it must be feasible[4]. The intuition, as explained earlier, is that IRA means that players expect that the negotiation may terminate after their offer, so it would be irrational to make an offer that would decrease their value of the game and hence their expected payoff.

3.2.3 Minimal offers and rationality thresholds

While feasibility is a necessary condition for an offer to be made in an PNG played under IRA, it is not sufficient for such offer to be part of an equilibrium strategy. Clearly, an optimal offer from one player to another should be a *minimal* feasible one, i.e., providing just a sufficient incentive for the recipient of the offer to play the desired action, but not more than that. The question of what is a minimal offer that achieves such objective

[4] As we have not defined 'rational solution', this is not a formal statement, so we cannot provide a proof.

crucially depends on the solution concept adopted by the recipient and used to determine his value of the game. Under our working assumption, such offer must increase the maximin value of the game for the recipient.

Thus, if a player A wants to induce with a preplay offer another player B to play a given strategy σ_B then, for any acceptable solution concept, it would suffice for A to make any sufficiently large offer that would turn σ into a *strictly dominant strategy* for B. But, such offer may be prohibitively costly or, depending on the solution concept and the rationality assumptions for B, unnecessarily high. For instance, when a player B receives an offer $A \xrightarrow{\delta/\sigma_B} B$, she should naturally expect that A (unless possibly bluffing) intends playing A's best response to σ_B. So, B can anticipate the outcome of the transformed game, and if B considers that outcome better than his current value, that should suffice for A's offer to work.

There is one technical caveat here. It is often the case that there is no *minimal* offer that guarantees to achieve the objective, e.g., to turn the desired strategy into a *strictly* dominant one. For instance, if it suffices for A to pay to B any amount that is *greater than* d for that purpose, then any offer of $d + \delta$, for $\delta > 0$, should do and there would be no optimal offer. Clearly, however, there is a practical minimum beyond which no player would consider it worth optimising any further. We will assume that such a sufficiently small **rationality threshold** $\epsilon > 0$, the same for all players, is fixed throughout the game and a common knowledge amongst the players. The threshold ϵ can be regarded as the cost of the recipient for considering and enforcing the offer. Therefore, in order to *ensure* that a player would consider the offer made to him and choose an action in favour of an outcome yielding him a payoff of d', rather than another one yielding him a payoff of only d, the difference $d' - d$ must be made at least ϵ. We can furthermore assume that ϵ is smaller than any positive difference between two payoffs in the starting NFG.

Thus, we adopt the notion of a **minimal (non-vacuous) offer** – one that guarantees an increase of the expected payoff of the recipient by exactly ϵ.

On the other hand, in order not to accumulate ϵ-deviations from optimal solutions in the course of the negotiation game or in the play of the resulting NFG and to keep our analysis simpler and neater, we will still assume, in accordance with the standard rationality assumptions, that a player who is to choose between several already available outcomes in the current NFG, would always choose to act in favour of the best one (or, any one of several equally best ones) for her, even if the difference with the second best outcome is below the threshold ϵ, but still positive.

3.2.4 Efficient negotiations and optimal plays

We will call an offer $A \xrightarrow{\delta/\sigma_B} B$ in a game Γ **efficient** if it is a minimal feasible offer, i.e. the values of Γ for both A and B are improved in the transformed game $\Gamma(A \xrightarrow{\delta/\sigma_B} B)$ and the increase of the value for B is at least ϵ.

Now, some important definitions. A strategy of a player in a PNG is an **efficient negotiation strategy** if:

1. all offers that it prescribes are efficient or vacuous,
2. it prescribes vacuous offers or pass only when no efficient offers are possible.
3. eventually it prescribes a pass, i.e., there is no infinite history in the PNG on every initial segment of which the strategy prescribes an offer.

An **efficient play of a PNG** is one where all players follow efficient negotiation strategies.

A play h of a PNG is **optimal**, under the adopted solution concept $\mathfrak{S}$, if any outcome in the solution $\mathfrak{S}(h(\mathcal{G}))$ of the transformed by h game $h(\mathcal{G})$ is maximal, i.e. is a redistribution of the vector of payoffs of a maximal outcome of the input game $\mathcal{G}$.

A remark on the last condition for efficiency of a negotiation strategy: it implicitly assume that the player would be better off eventually playing the (transformed) NFG rather than procrastinating forever. That need not be the case if all outcomes in the player's solution of every transformed NFG obtained in the negotiation phase yield negative payoffs (i.e., loss) for the player. If that is the case for both players, then they would simply agree not to play the game if given such option, else the negotiation phase would have to be terminated exogenously, say when the time for negotiations runs out. If that is the case for only one of the players, however, then the other will sooner or later pass or opt out, thus preventing an infinite preplay negotiation.

4 Efficient plays of 2-player preplay negotiation games

Here I will develop a method for determining the *best* (for the offerer) efficient offers that a player can make in a given 2-player PNG and illustrate with some examples possible evolutions and outcomes of efficient plays of preplay negotiation games and will draw some conclusions.

For presenting and illustrating the method, hereafter I adopt the solution concept $\mathfrak{S}$, prescribing as solution for any normal form game $\mathcal{G}$ the set of outcomes generated by pure strategy Nash equilibria of the game, if such exist, otherwise the set of outcomes generated by the maximin strategies of the players. As we will see further, the latter case will only possibly apply to the input NFG, because from the first offer on every current NFG will have a pure strategy Nash equilibrium.

4.1 Computing the best efficient offers of a player

What is an IRA-based rational player's reasoning when considering making an unconditional and irrevocable offer to another player in a given NFG $\mathcal{G}$? Suppose, player A considers making such an offer to player B. (The reasoning when B is to make the offer is completely symmetric.) Then, for each strategy B_j of B, player A considers making an offer contingent on B playing B_j. To make sure that B will play B_j in the resulting NFG, it suffices to make B_j a strictly dominant strategy for B. The necessary payment for that, however, can be unreasonably high for A because, after that payment, A's best response to B_j may yield a worse payoff than the current (e.g., maximin) value for A of the original game. So, a more subtle reasoning is needed, presented by the following procedure.

1. For each strategy B_j of B, player A looks at her best response to B_j. Suppose for now that it is unique, say A_{i_j}. Then, this is what B should expect A to play if B knows that A expects B to play B_j. In this case, A computes the minimal payment needed to make B_j not necessarily a strictly dominant strategy, but a *best response to* A_{i_j}, i.e., the minimal payment that would make the strategy profile $\sigma_{i_j,j} = (A_{i_j}, B_j)$ a Nash equilibrium. That payment is

$$\delta^A_{i_j,j} = \max_k (u_B(A_{i_j}, B_k) - u_B(\sigma_{i_j,j})).$$

   Clearly, $\delta^A_{i_j,j} \geq 0$. Suppose it is positive, or is 0 but is reached for more than one values of k. Then, in order to break B's indifference and make $\sigma_{i_j,j}$ a *strict* Nash equilibrium, A has to add to $\delta^A_{i_j,j}$ a threshold amount ϵ, thus eventually producing the minimal necessary payment $\delta^A_j = \delta^A_{i_j,j} + \epsilon$.
2. If A's best response to B_j is not unique, then A must compute the minimal payment δ^A_j needed to make B_j the best response of B to *each* of A's best responses to B_j. Clearly, that is the maximum of all $\delta^A_{i_j,j}$ computed above, possibly plus ϵ.
3. Once δ^A_j is computed, A computes her payoff in the transformed game $\widehat{\mathcal{G}}_{B_j}$ after an offer $A \xrightarrow{\delta^A_j/B_j} B$ in the outcome $\sigma_{i_j,j}$, which is $v^A(\widehat{\mathcal{G}}_{B_j}) = u_A(\sigma_{i_j,j}) - \delta^A_j$.
4. Finally, A maximizes over j:

$$v^A(\widehat{\mathcal{G}}) = \max_j v^A(\widehat{\mathcal{G}}_{B_j}).$$

   Now, there are 4 cases to consider:

   (a) If $\delta^A_j > 0$ and the maximum $v^A(\widehat{\mathcal{G}})$ is achieved for only one j, then the best efficient offer of A is determined as

$$A \xrightarrow{\delta^A_j/B_j} B.$$

   (b) If $\delta^A_j > 0$ and the maximum $v^A(\widehat{\mathcal{G}})$ is achieved for more than one j, then A can choose any of these and compute her best efficient offer as above. Better still, A can choose the j yielding the *least* payoff for B, thus stimulating B to make her a further offer.

   (c) If $\delta^A_j = 0$ and $v^A(\widehat{\mathcal{G}})$ is reached for only one value of j, then there is no need for A to make any offer, because in this case there is a unique dominant strategy Nash equilibrium in the game, so A cannot make any offer that would improve on her payoff yielded by that Nash equilibrium. In this case, the game is considered already solved in the optimal way that it can be achieved by using preplay offers.

(d) If $\delta_j^A = 0$ and $v^A(\widehat{\mathcal{G}})$ is reached for more than one values of j, then there are several dominant strategy Nash equilibria in the game, so A must still make a vacuous offer $A \xrightarrow{0/B_j} B$ in order to indicate to B for which Nash equilibrium she will play.

It is now up to player A to decide whether to make the respective offer leading to the value $v^A(\widehat{\mathcal{G}})$ – if that offer would improve her current value – or not, otherwise. The available alternatives to A are to pass, thus possibly ending the negotiations, or to make just a vacuous offer, for the sake of indicating to B on which of the several equivalent Nash equilibria to coordinate (as in the symmetric coordination game), when appropriate.

The procedure outlined above[5] implies the following claim.

Theorem 1 *Given any input NFG $\mathcal{G}$, let $A \xrightarrow{\delta_j^A/B_j} B$ be a best efficient offer of A to B in $\mathcal{G}$, as computed by the procedure described above. Then:*

1. *The resulting transformed game has a pure strategy Nash equilibrium $\sigma_{i_j,j} = (A_{i_j}, B_j)$.*
2. *The value for the player A of that transformed game is $v^A(\widehat{\mathcal{G}})$.*
3. *$v^A(\widehat{\mathcal{G}})$ is the best value for the player A of a transformed game induced by an efficient offer from A to B in $\mathcal{G}$.*

Corollary 2 *Any non-dominated efficient negotiation strategy for player A in a given PNG prescribes only making best efficient offers or vacuous offers, or passing.*

4.2 Solving strategic games by exchange of best efficient offers

Here we will give some examples illustrating the method. We adopt the following notation: $d^+ := d + \epsilon$ and $d^- := d - \epsilon$.

4.2.1 The power of best efficient offers

Example 3 (Perfectly solving a game by an efficient and optimal play)
Consider the following NFG $\mathcal{G}$ between players R (row) and C (column):

	$C1$	$C2$	$C3$
$R1$	$2, 10$	$10, 4$	$5, 1$
$R2$	$6, 0$	$4, 4$	$6, 3$

This game has no pure strategy Nash equilibria and the only dominated strategy is $C3$. The only maximin solution is $(R2, C2)$ with payoffs $(4, 4)$. Hence, the value of this game for each of the players R and C is 4.

Suppose, player R is to make the first offer. Let us compute the best efficient offer that R can make to C.

[5] This procedure for computing best efficient offers and the execution of preplay negotiations based on them have been implemented by François Schwarzentruber and is available online on `http://people.irisa.fr/Francois.Schwarzentruber/negotiationgames`.

- *The best response of R to $C1$ is $R2$. So,* $\delta^R_{2,1} = 4 - 0 + \epsilon = 4^+$ *and* $v^R(\widehat{\mathcal{G}}_{C1}) = 6 - 4^+ = 2^-$
- *The best response of R to $C2$ is $R1$. So,* $\delta^R_{1,2} = 10 - 4 + \epsilon = 6^+$ *and* $v^R(\widehat{\mathcal{G}}_{C2}) = 10 - 6^+ = 4^-$.
- *The best response of R to $C3$ is $R2$. So,* $\delta^R_{2,3} = 4 - 3 + \epsilon = 1^+$ *and* $v^R(\widehat{\mathcal{G}}_{C3}) = 6 - 1^+ = 5^-$.

Thus, $v^R(\widehat{\mathcal{G}}) = v^R(\widehat{\mathcal{G}}_{C3}) = 5^-$*, meaning that R's best offer to C is* $R \xrightarrow{1^+ \ / \ C3} C$. *The resulting transformed game is*

	$C1$	$C2$	$C3$
$R1$	$2, 10$	$10, 4$	$4^-, 2^+$
$R2$	$6, 0$	$4, 4$	$5^-, 4^+$

It has one Nash equilibrium $(R2, C3)$, created by R's offer and yielding a Pareto optimal outcome with payoffs $(5^-, 4^+)$ which are the values of the players for this game. They are strictly better than their previous values and, moreover, the offer by C also serves as a signalling for R towards that equilibrium. However, the outcome $(R2, C3)$ is not yet maximal.

The best efficient offer of C to to R in the transformed game can be computed likewise (see details in the full paper [3]) to be $C \xrightarrow{4^+ \ / \ R1} R$. *Moreover,* $v^C(\widehat{\mathcal{G}}) = 6^-$*, which is better than C's current value of 4^+, so C can improve his value of the game by making that offer. The resulting transformed game is*

	$C1$	$C2$	$C3$
$R1$	$6^+, 6^-$	$14^+, 0^-$	$8, -2$
$R2$	$6, 0$	$4, 4$	$5^-, 4^+$

It has one Nash equilibrium $(R1, C1)$, which is Pareto optimal. The strategy $R1$ is strictly dominant for R, yielding payoffs $(6^+, 6^-)$ which are the values of the players for this game. They are strictly better than the previous ones of $(5^-, 4^+)$, but not yet maximal. R's best offer to C now is $R \xrightarrow{6^+ \ / \ C2} C$ *and* $v^R(\widehat{\mathcal{G}}) = v^R(\widehat{\mathcal{G}}_{C2}) = 8$*, which is better than R's current value of 6^+, so R should make that offer. The resulting transformed game is*

	$C1$	$C2$	$C3$
$R1$	$6^+, 6^-$	$8, 6$	$8, -2$
$R2$	$6, 0$	$-2^-, 10^+$	$5^-, 4^+$

It has a strictly dominant strategies equilibrium $(R1, C2)$ yielding payoffs $(8, 6)$ which are the values of the players for this game. They are strictly better than the previous ones $(6^+, 6^-)$. In fact, this is the (only) maximal outcome in the game, and one can now check that none of the players can make any further improving offers. Thus, the game is now perfectly solved and this is the outcome of the negotiation phase.

I leave to the reader to check that if C makes the first offer the negotiation phase will end after each player making only one offer and with a slightly different game but with the same solution. As we will see further, such confluence is not always the case.

4.2.2 The possible weakness of effective offers

The example above demonstrates the potential power of efficient offers to solve normal form games. On the other hand, the version of the Prisoners' Dilemma game in Figure 2.3 demonstrate their weakness, showing that in preplay negotiation games where no conditional offers and no withdrawals are allowed the players may be unable to reach any Pareto optimal outcome by means of exchanging efficient preplay offers. Moreover, the value of the game that a player can achieve by making any *effective* offer in such a preplay negotiations game, can be worse than the original value of the game for *every* player, as demonstrated by the following example.

Example 4 (No player benefits by making an effective offer)
Consider the following game $\mathcal{G}$ between players R (row) and C (column):

	$C1$	$C2$
$R1$	$3,3$	$2,2$
$R2$	$9,1$	$0,8$
$R3$	$0,7$	$8,1$

Again, the game has no pure strategy Nash equilibria and the only maximin outcome is $(R1,C2)$ with payoffs $(2,2)$, which is not Pareto optimal. The players have the potential to negotiate a mutually better deal in any of the outcomes in rows 2 and 3. However, it turns out that none of them can make a first efficient offer that would improve her expected payoff. Indeed, one can compute that $v^R(\widehat{\mathcal{G}}) = 2^-$ and $v^C(\widehat{\mathcal{G}}) = 0^-$. Both values are less than the respective maximin values of 2. Therefore, no player is interested in making a first offer and the negotiation phase ends at start, after each player passes.

Thus, we have the following observation.

Proposition 5 *Not every normal form game can be solved optimally by an efficient play of the preplay negotiation game with unconditional and irrevocable offers.*

4.2.3 The disadvantage of making the first offer

Even when each of the players can start an effective negotiation ending with a solved game, the solution may essentially depend on who makes the first effective offer, as shown by the next example.

Example 6 *Consider the following game between R and C:*

	$C1$	$C2$
$R1$	$1,8$	$10,4$
$R2$	$4,10$	$1,11$
$R3$	$4,0$	$2,2$

One can check that if the first offer is made by R the preplay negotiation game ends with the game below on the left: where the only Nash equilibrium outcome is $(R1,C2)$.

	$C1$	$C2$
$R1$	$1, 8$	$6^-, 8^+$
$R2$	$4, 10$	$-3^-, 15^+$
$R3$	$4, 0$	$-2^-, 6^+$

	$C1$	$C2$
$R1$	$4^+, 5^-$	$9, 5$
$R2$	$4, 10$	$-3^-, 15^+$
$R3$	$4, 0$	$-2^-, 6^+$

Figure 2: The transformed games after first offer by R (on the left) and by C (on the right)

(It is also the only acceptable outcome, surviving iterated elimination of strictly dominated strategies). It yields payoffs $(6^-, 8^+)$*, which are the values of the game for the players. Likewise, if* C *makes the first offer, the preplay negotiation game ends with the game on the right, where the only acceptable outcome is again* $(R1, C2)$*, but now yielding payoffs* $(9, 5)$*. In both cases the disadvantaged player is the one who has made the first offer.*

The example above also indicates that the greedy approach, where a player always makes the best effective offer he can, may not be his best long term strategy. Passing the turn to the other player – that is, making a vacuous offer – could be strategically more beneficial. On the other hand, if both players keep exchanging only vacuous offers or passing, then they will never improve their values of the starting game. Yet, one can check in the example above that any pair of strategies such that:

one player takes the initiative by making the first effective move with his best first offer and thereafter always responds with his currently best effective offers until possible and then passes,

while

the other player remains passive (makes only vacuous offers) until the first effective offer is made, and thereafter keeps responding with her best offers until possible and then passes,

is a subgame-perfect equilibrium strategy in the preplay negotiation for that game.

5 Concluding remarks and further agenda

This paper has demonstrated that preplay negotiations where the players can exchange unilaterally binding and irrevocable offers for payments of incentives to their opponent under the Immediate Rationality Assumption can be a powerful means for achieving better for all players outcomes in the resulting transformed normal form games, but on the other hand their bargaining powers to achieve their *best* outcomes in such games can be substantially affected by the potential disadvantage of making the first effective offer in such games. The analysis of the general n-player case is much more complicated than the 2-players case, as it involves embedded coalitional games between players interested in making collective offer to other players, while keeping in mind that the transformed normal form game remains a non-cooperative game where every player eventually plays for themselves. This analysis is left to a future work.

Acknowledgments

I acknowledge the contribution of Paolo Turrini to the initial stage of this research, in the context of our joint work published in [5] and [6]. I thank the referees for some helpful comments.

References

[1] Tore Ellingsen and Elena Paltseva. Non-cooperative contracting. Working paper, 2011, `http://www2.hhs.se/personal/ellingsen/pdf/Non-cooperativeContracting5.pdf`.

[2] Valentin Goranko. Transformations of normal form games by preplay offers for transfer of utility. Working paper, 2012, `http://arxiv.org/abs/1208.1758`.

[3] Valentin Goranko. Preplay negotiations by unconditional offers in 2-player strategic games: power and weakness. Working paper, `https://www.dropbox.com/s/bszw38dngrh9y3i/Goranko-PreplayNegotiationsUnconditionalOffers.pdf`, 2016.

[4] Valentin Goranko. Extensive games with interplay offers for incentive payments: an emergence of non-cooperative cooperation. Working paper, to be presented at GAMES 2016, `https://www.dropbox.com/s/p0dsqqg4bwgj0yg/Goranko-Extensive-Games-with-Interlay-offers-2016-02-12.pdf`.

[5] Valentin Goranko and Paolo Turrini. An offer you cannot refuse: obtaining efficiency and fairness in preplay negotiation games with conditional offers. In Huaxin Huang, Davide Grossi, and Olivier Roy, editors, *Proc. of the Fourth International Workshop on Logic, Rationality and Interaction (LORI'2013)*, volume 8196 of *LNCS*, pages 110–123. Springer, 2013.

[6] Valentin Goranko and Paolo Turrini. Two-player preplay negotiation games with conditional offers. *International Game Theory Review*, 18(1), `http://www.worldscientific.com/doi/abs/10.1142/S0219198915500176`, 2016.

[7] Joel M. Guttman. Understanding collective action: Matching behavior. *American Economic Review*, 68(2):251–255, 1978.

[8] Joel M. Guttman. A non-Cournot model of voluntary collective action. *Economica*, 54(213):1–19, 1987.

[9] Matthew O. Jackson and Simon Wilkie. Endogenous games and mechanisms: Side payments among players. *Review of Economic Studies*, 72(2):543–566, 2005.

[10] Martin Osborne and Ariel Rubinstein. *A Course in Game Theory.* MIT Press, 1994.

[11] A. Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica*, 50:97–110, 1982.

[12] Thomas Schelling. *The strategy of conflict.* Harvard University Press, 1960.

[13] Ilya Segal. Contracting with externalities. *Quarterly Journal of Economics*, 114(2):337–388, 1999.

[14] Akira Yamada. Alternate contracts for side payments. *Economics Bulletin*, 3(46):1–5, 2005.